Thesis Summary

Mechanical Behaviour of Rock Materials under Dynamic Loading

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Abstract

The deformation and fracture behaviours of rocks are of significance for analysing the safety and long term stability of rock engineering structures when subjected to various loads. Mechanical behaviours of rock materials are different under different loading conditions. In this thesis, combined experimental and analytical approaches are developed to investigate various aspects of mechanical behaviour over a wide range of strain rates, from quasi-static to impact loadings.

A critical review is carried out to identify the validity and applicability of experimental techniques (i.e. loading techniques, optical measurement techniques and testing methods), and to present the state-of-the-art in mechanical behaviour (including mechanical properties, physical mechanisms of strain rate, rate-dependent constitutive models, and fracture criteria) of rock materials under dynamic loading.

Four types of testing methods have been developed using a modified split Hopkinson pressure bar system; notched semi-circular bending method to determine the mode-I crack initiation and propagation toughness and fracture energy; Brazilian disc method to determine the tensile strength; uniaxial compression tests to determine the uniaxial compressive strength; and passive confining pressure method to determine the triaxial compressive strength. The digital image correlation method in conjunction with high-speed photography is developed for the measurement of full-field strain fields and crack opening displacement. Three sets of shock wave experiments have been conducted to determine the shock Hugoniot behaviour, spall tensile strength and shear strength using a plate impact facility. Embedded manganin stress gauges and a high-speed velocimeter using heterodyne techniques are used for the measurements of stresses, shock velocity and free surface velocity. The detailed experimental procedures used to make measurements are to great extent standard methods when determining mechanical properties and deformation fields under dynamic loadings.

To systematically study the effect of strain rate, well-studied igneous, sedimentary, and metamorphic rocks in dynamic experiments are selected in this research work. The normalized dynamic strength and fracture toughness (i.e. the ratio of dynamic strength or toughness to the quasi-static one) of fine-grained marble and gabbro are apparently higher than that of coarse-grained marble and sandstone, which are mainly governed by the following factors: microstructures, wave speeds, fracturing process, and induced microcrack features.

Micro-fractographic studies are carried out to characterize failure mechanisms using a scanning electron microscope and a three-dimensional optical profilometry. The operating mechanisms of dynamic failure are mostly of transgranular microcracks, and moreover, with the increase of loading rate, the number of some typical microcrack modes, such as multiple cleavage steps, opening microcracks and microbranches, is increased. The fractal dimension is dependent to a large extent on microcrack modes. Based on macro- and microscopic experimental results and energy-based fracture mechanics, a micromechanical mode is proposed to explain the failure transition from intergranular fracture to transgranular fracture. The micromechanical model is also successful for predicting the fracture toughness of the grain and the grain boundary under quasi-static and dynamic loadings.

This thesis critically reviews the state-of-the-art of experimental techniques and mechanical behaviour of rock materials under dynamic loading. New results of experimental tests and analytical investigations are presented to gain an in-depth understanding of rock dynamic problems in terms of stress wave propagation and dynamic fracture.

Keywords: Mechanical behaviour, dynamic loading, high strain rate, failure mechanisms, fracturing modes
1 Introduction

1.1 Overview of Rock Dynamics

Rock mechanics was defined by the US National Committee on Rock Mechanics in 1964 and subsequently modified in 1974: ‘Rock mechanics is the theoretical and applied science of the mechanical behaviour of rock and rock masses; it is that branch of mechanics concerned with the response of rock and rock masses to the force fields of their physical environment.’ Within the field of rock mechanics, an important subcategory is rock dynamics, dealing with the mechanical behaviour of rock materials and rock masses under dynamic loading conditions, where an increased rate of loading induces a change in mechanical properties and fracture behaviour. Rock dynamic covers a wide scope ranging from the initiation and forms of dynamic loads, transmission and attenuation of stress waves, rock dynamic damage evolution and fracturing, to the support of rock structures subjected to dynamic loads. Figure 1 illustrates typical rock dynamics issues related to the construction and utilization of an underground cavern. As schematically shown in Figure 2, there are four zones around an underground nuclear explosion. The vaporized and melted zones are in the category of earth science at extremely high pressure, in which they will not be covered by the present study.

Figure 1 Overview of the rock dynamic problems and influencing factors in an underground engineering design

Figure 2 Schematic of an underground nuclear explosion: (I: Vaporized, II: Melted, III: Crashed, and IV: Damaged Zones, data from Butkovich, 1965)
Dynamic behaviour of materials has been extensively investigated since the early 1900’s (Hopkinson 1901; Hopkinson 1914). However, it should be mentioned that guidance and standards in dynamic testing and design are generally lacking. The following issues remain to be addressed:

- Some inherent technical issues of dynamic experimental techniques and their validity and applicability need to be carefully identified.
- It is required to critically assess the state-of-the-art of numerous dynamic tests.
- Accurate determination of mechanical properties over a wide range of strain rates is important to develop the rate-dependent constitutive models and fracture criteria, and to validate numerical simulations.
- Novel real-time optical methods, especially in combination with high-speed photography are extremely necessary to perform high-rate deformation measurement accurately and quantitatively.
- Microscopic experimental and theoretical investigations are required to study the physical mechanisms of strain-rate effect.

Therefore, a comprehensive knowledge would benefit from quantitatively experimental and further theoretical approaches, and also is essential for research in leading to a deeper understanding of rock dynamics.

1.2 Objectives

The aim of this thesis is to perform the combined quantitatively experimental and theoretical methods gaining further knowledge of dynamic mechanical behaviour of rocks. It is important to design reliable experimental methods, to determine mechanical properties, to measure full-field deformation field, and to investigate failure mechanisms of rock materials at laboratory scale. More specifically, the main objectives are classified into the following categories:

- **A Critical Review of Dynamic Experimental Techniques and Mechanical Behaviour:** Various types of loading techniques, optical measurement techniques, testing methods, and associated inherent technical issues are analysed and their applications to rock materials are critically reviewed. In-depth discussions are given on dynamic mechanical properties, empirical rate-dependent equations, physical mechanisms of strain rate effect, phenomenological and mechanically-based rate-dependent constitutive models, and dynamic fracture criteria.

- **Quantitative Determination of Dynamic Mechanical Properties:** An accurate knowledge of material properties is of significance in rock engineering design. Dynamic mechanical properties of rock materials (including dynamic uniaxial and triaxial compressive strength, splitting tensile strength, mode-I crack initiation and propagation toughness, fracture energy, kinetic energy of fragments, Hugoniot elastic limit, spall tensile strength, shear strength, etc.) are quantitatively obtained by a modified SHPB and a plate impact facility.

- **High-speed Optical Measurement Techniques:** The digital image correlation (DIC) method in conjunction with high-speed photography is developed for the measurement of full-field strain fields and crack opening displacement in the SHPB tests. A high-speed velocimeter using heterodyne technique (HetV) is employed to determine the free surface velocity in the plate impact experiments.

- **Micromeasurement and Micromechanical Modelling of Failure Mechanisms:** Systematic micro-fractographic studies on fracture surfaces are carried out, and a micromechanical model is established to examine failure micromechanisms under quasi-static and dynamic loads.
2 Experimental Techniques for Dynamic Testing

2.1 Loading Techniques

A classification of loading techniques and mechanical states for rocks over a wide range of strain rates is shown in Figure 3, which is particularly based on experimental results of rock materials. Mechanical properties and fracture behaviours exhibit a general trend: i.e. they change with the loading rate. In particular, the responses distinguishably change after the loading rate exceeds a critical value. Two reviews of 50 years of research since the pioneering work in the 1960s is intended to present the state of the art in experimental techniques for the determination of dynamic behaviour of rock materials ranging from $10^{-1}$ to $10^{6}$ s$^{-1}$.

**Figure 3** Classification of loading techniques and the state of rock materials over a wide range of strain rates

One of the most widely used loading techniques for HSR is the split Hopkinson bar (SHB) or the Kolsky bar. The principle of the traditional SHPB technique is briefly described in this section. We focus on the key techniques for characterizing the dynamic response of rock materials, as summarized in Table 1.

Table 1 Major developments in split Hopkinson bar for rock materials

<table>
<thead>
<tr>
<th>Year</th>
<th>Major developments</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>1966</td>
<td>Stress-strain relation</td>
<td>(Hauser 1966)</td>
</tr>
<tr>
<td>1967</td>
<td>The first thesis: stress-strain; size effect; energy transmission and failure characteristics</td>
<td>(Hakalehto 1967)</td>
</tr>
<tr>
<td>1968</td>
<td>Temperature effect</td>
<td>(Kumar 1968)</td>
</tr>
<tr>
<td>1968</td>
<td>High-speed camera for recording dynamic fracturing</td>
<td>(Perkins and Green 1968)</td>
</tr>
<tr>
<td>1970</td>
<td>Stress-strain curve; temperature effect</td>
<td>(Perkins et al. 1970)</td>
</tr>
<tr>
<td>1972</td>
<td>Hydrostatic confining pressure chamber; stress-strain curves; truncated cone striker</td>
<td>(Christensen et al. 1972)</td>
</tr>
<tr>
<td>1974</td>
<td>Anisotropy effect; cylindro-conical striker; direct tension test</td>
<td>(Howe et al. 1974)</td>
</tr>
<tr>
<td>1974</td>
<td>Effects of confining pressure and temperature; hydrostatic confining pressure chamber</td>
<td>(Lindholm et al. 1974)</td>
</tr>
<tr>
<td>1976</td>
<td>Anisotropy effect; direct tension and torsion tests</td>
<td>(Goldsmith et al. 1976)</td>
</tr>
<tr>
<td>1977</td>
<td>Pure shear test using the TSHB and a thin-walled tubular specimen</td>
<td>(Lipkin et al. 1977)</td>
</tr>
<tr>
<td>1979</td>
<td>The radial inertia effects</td>
<td>(Powell 1979)</td>
</tr>
<tr>
<td>1984</td>
<td>WLCT method for fracture toughness</td>
<td>(Klepaczko et al. 1984)</td>
</tr>
<tr>
<td>1987</td>
<td>Spalling test</td>
<td>(Khan and Irani 1987)</td>
</tr>
<tr>
<td>1990</td>
<td>SENB method for fracture toughness; an optical technique for measuring COD</td>
<td>(Tang and Xu 1990)</td>
</tr>
<tr>
<td>1993</td>
<td>BD method for tensile strength</td>
<td>(Dutta and Kim 1993)</td>
</tr>
<tr>
<td>1993</td>
<td>Effects of stress waveforms produced by rams on energy dissipation and fragmentation</td>
<td>(Li et al. 1993)</td>
</tr>
<tr>
<td>1994</td>
<td>Saturation effect in the spalling test</td>
<td>(Lou 1994)</td>
</tr>
<tr>
<td>1995</td>
<td>Dynamic Moiré method for detecting the time-to-fracture and measuring COD</td>
<td>(Yu and Zhang 1995)</td>
</tr>
</tbody>
</table>
In 1997, inverse analysis for stress-strain curve; separation of stress waves (Zhao and Gary 1997) was conducted. In 1999, the SR method for fracture toughness; temperature effect; fracture characteristics; energy (Zhang et al. 1999) was introduced. In 2000, a truncated-cone shaped striker; oscillation elimination (Li et al. 2000) was developed. In 2001, pulse shaper; size effect (Frew et al. 2001) was studied. In 2005, 75 mm diameter bar; intermediate strain rate; fracture modes; energy (Li et al. 2005) was explored. In 2006, FBD method for tensile strength and elastic modulus (Wang et al. 2006) was introduced. In 2007, wave propagation through fractured rocks with fractal joint surfaces (Ju et al. 2007) was investigated. In 2009, infrared thermography for measuring temperature (Shi et al. 2009) was developed. In 2010, triaxial SHPB; hydrostatic confining pressure chamber (Frew et al. 2010) was introduced. In 2011, punch shear method for shear strength (Huang et al. 2011) was developed. In 2012, the designed triaxially compressed split Hopkinson bar (TriSHB) (Cadoni and Albertini 2012) was introduced. In 2013, HS-DIC for UC, BD and NSCB testing (Zhang and Zhao 2013) was developed.

2.2 Measurement Techniques

The ability to quantitatively obtain dynamic mechanical properties and deformation fields, to fully understand various fracturing process and failure mechanisms and to effectively validate theoretical models of material behaviour is largely dependent on measurement techniques.

2.3 Testing Methods

Suggested methods from the ISRM and ASTM for determining mechanical properties of rock materials under quasi-static loading are based on core-shaped samples, since such specimens can be easily prepared. Most dynamic testing methods are extended or modified from quasi-static ones, as summarized in Table 2, which also include measurement techniques and methods for interpreting experimental data.

2.4 Summary

This review of 50 years of research since the pioneering work in the 1960s is intended to present the state of the art in experimental techniques for the determination of dynamic behaviour of rock materials. The full understanding of dynamic mechanical behaviour heavily depends on reliable experimental techniques, testing procedures and effective numerical calibrations. This chapter presents some prospects requiring further investigation.
Table 2 Summary of testing methods for determining mechanical properties of rock materials under both quasi-static and dynamic loading conditions

<table>
<thead>
<tr>
<th>Loading type</th>
<th>Testing methods</th>
<th>Quasi-static properties</th>
<th>Dynamic properties</th>
<th>Calculation for</th>
<th>Determination of</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tension</strong></td>
<td>DT</td>
<td>$\sigma_t$ (ASTM 2008a)</td>
<td>$\sigma_{sl}$ (Goldsmith et al. 1976)</td>
<td>$\sigma_{sl}$ (Zhao and Li 2000)</td>
<td>Pressure transducer or SG on bars</td>
</tr>
<tr>
<td></td>
<td>BD</td>
<td>$\sigma_t$ (ISRM 1978)</td>
<td></td>
<td></td>
<td>Peak load</td>
</tr>
<tr>
<td></td>
<td>FBD</td>
<td>$\sigma_t$ (Wang et al. 2004)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>UC</td>
<td>$\sigma_{uc}$ (ISRM 1979)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>TC</td>
<td>$\sigma_{tc}$ (ISRM 1983)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Shear</td>
<td>$\tau$ (Stacey 1980)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Compression</strong></td>
<td>HC(F)BD</td>
<td>$K_c$ (Fischer et al. 1996)</td>
<td>$K_{id}$ (Wang et al. 2010)</td>
<td>Finite element method and SG</td>
<td>Dynamic Moiré</td>
</tr>
<tr>
<td></td>
<td>CST(F)BD</td>
<td>$K_{ic}$, $K_{isc}$ (Atkinson et al. 1982)</td>
<td>$K_{id}$, $K_{id}$ (Nakano et al. 1994)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SR</td>
<td>$K_c$ (Ouchterlony 1988)</td>
<td>$K_{id}$ (Zhang et al. 2000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$K_{id}$ (Costin 1981)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>WLCT</td>
<td>$K_c$ (Klepaczko et al. 1984)</td>
<td>$K_{id}$ (Klepaczko et al. 1984)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>HCBD</td>
<td>$K_c$ (Fischer et al. 1996)</td>
<td>$K_{id}$ (Lambert and Ross 2000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>CCNBBD</td>
<td>$K_c$ (Fowell 1995)</td>
<td>$K_{id}$ (Dai et al. 2010a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Bending</strong></td>
<td>TPB</td>
<td>$\sigma_t$ (Jaeger 1967)</td>
<td>$\sigma_{sl}$ (Zhao and Li 2000)</td>
<td>Quasi-static theory, FEM calibrating $f(a/R)$</td>
<td>Peak load</td>
</tr>
<tr>
<td></td>
<td>SCB</td>
<td>$\sigma_t$ (Van de Ven et al. 1997)</td>
<td>$\sigma_{sl}$ (Dai et al. 2008)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SENB</td>
<td>$K_c$ (ASTM 2011)</td>
<td>$K_{id}$ (Zhao et al. 1999b)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>CNSCB</td>
<td>$K_c$ (Kuruppu 1997)</td>
<td>$K_{id}$ (Dai et al. 2011)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>NSCB</td>
<td>$K_c$ (Chong and Kuruppu 1984; Kuruppu et al. 2013)</td>
<td>$K_{id}$ (Chen et al. 2009)</td>
<td>Thermodynamics + LGG</td>
<td>LGG, HS-camera</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$K_{id}$ (Chen et al. 2009)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$K_{id}$ (Zhang and Zhao 2013a)</td>
<td>Quasi-static theory</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$K_{id}$ (Zhang and Zhao 2013b)</td>
<td>Thermodynamics + DIC</td>
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<td></td>
</tr>
</tbody>
</table>
3 Dynamic Mechanical Behaviour of Rock Materials

3.1 Dynamic Uniaxial Compressive Behaviour

Figure 4 shows the results for the normalized uniaxial compressive strength as a function of strain rate obtained over the last five decades.

![Figure 4 Normalized dynamic uniaxial compressive strength as a function of strain rate](image)

3.2 Dynamic Tensile Behaviour

Figure 5 shows the normalized dynamic tensile strength as a function of loading rate. It can be seen that the strain rates of the DT results are higher than those obtained by indirect tension testing methods, since the specimen sizes are usually smaller in indirect tension tests.

3.3 Dynamic Fracture Behaviour

The dynamic crack initiation toughness $K_{id}$ is determined by the time to fracture $t_i$ and is given by the equation $K_{id} = K_{id}^{th} (t_i)$. Figure 6 shows the normalized dynamic crack initiation toughness as a function of normalized loading rate for several materials.
3.4 Semi-Empirical Equations for Rate Dependent Strength

There have been many attempts to derive semi-empirical equations to express the relationship between strain/loading rate and the mechanical properties of rock materials, as summarized in Table 3.

<table>
<thead>
<tr>
<th>Rock type</th>
<th>Strain rate ( (s^{-1}) )</th>
<th>Semi-empirical equation</th>
<th>Material constants</th>
<th>Refs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Granite</td>
<td>( 10^5 - 10^4 )</td>
<td>( \sigma_{ud} = C \log(\dot{\varepsilon}) + \sigma_{uc} )</td>
<td>( C = 13 ), ( \sigma_{uc} = 340 )</td>
<td>(Masuda et al. 1987)</td>
</tr>
<tr>
<td>Granite</td>
<td>( 10^4 - 10^5 )</td>
<td>( \dot{\varepsilon}_t = 0.5 ) \text{MPa/s} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rocks</td>
<td>( 10^6 - 10^7 )</td>
<td>( \sigma_{ud} \propto \dot{\varepsilon}^{1.1 + n} ), ( \dot{\varepsilon} &lt; 10^7 ); ( \sigma_{ud} \propto \dot{\varepsilon}^{1/n} ), ( \dot{\varepsilon} \geq 10^7 )</td>
<td>( a ), ( n_{\text{max}} = 1/3 )</td>
<td>(Grady and Lipkin 1980)</td>
</tr>
<tr>
<td>Limestone</td>
<td>( 10^6 - 10^7 )</td>
<td>( \sigma_{ud} \propto \dot{\varepsilon}^{0.007} ), ( \dot{\varepsilon} &lt; 76s^{-1} ); ( \sigma_{ud} \propto \dot{\varepsilon}^{0.35} ), ( \dot{\varepsilon} &gt; 76s^{-1} )</td>
<td>( n_c = 130 ), ( n = 0.3 )</td>
<td>(Lankford 1981)</td>
</tr>
<tr>
<td>Tuff</td>
<td>( 10^6 - 10^7 )</td>
<td>( \sigma_{td} = k_u e^{u/(1+q)} \left( \sigma_{tc} - \sigma_{tc}^{1/2} \right) )</td>
<td></td>
<td>(Olsson 1991)</td>
</tr>
<tr>
<td>Limestone</td>
<td>( 10^5 - 10^4 )</td>
<td>( \sigma_{td} = k_u e^{u/(1+q)} \left( \sigma_{tc} - \sigma_{tc}^{1/2} \right) )</td>
<td>( U ), ( k_1 = 40 )</td>
<td>(Serdengecti and Boozer 1985)</td>
</tr>
</tbody>
</table>
3.5 Dynamic Fracture Criteria

A summary of dynamic fracture criteria and a brief description to rock materials are presented in Table 4.

![Table 4 A summary of dynamic fracture criteria for quasi-brittle materials](image)

3.6 Summary

This review of 50 years of research presents the state of the art in dynamic behaviour of rock materials. The full understanding of dynamic behaviour depends on reliable experimental techniques, testing procedures and effective numerical simulations.
4 Dynamic Mechanical Properties and Full-Field Strain Measurements

4.1 Material Characterizations

Figure 7 shows the scanned images of thin-section at a resolution of 2,400 dpi and the optical cross-polarized micrographs of four types of rock.

Figure 7 Scanned images of thin-section at a resolution of 2,400 dpi (top) and cross-polarized micrographs (bottom) of: (a) sandstone, (b) gabbro, (c) coarse-grained marble, (d) fine-grained marble

4.2 Split Hopkinson Pressure Bar

The SHPB consists of a striker bar, an incident bar and a transmission bar, with a specimen sandwiched between the incident and transmission bars, as shown in Figure 8. Figure 9 shows the photos of the modified SHPB system. The SHPB system consisted of a gas gun, a cone-shaped striker, an incident bar, a transmission bar, a momentum bar and a momentum trap. The striking velocity was measured by a laser-beam velocity measurement system. The high-speed imaging system consists of a CMOS sensor-based HS-camera (Photron Fastcam SA1.1), a macro-lens (Kenko PRO 300 2.0× objective lens), a set of extension tube (Kenko 12, 20 and 36 mm), and a ring-shaped flash light (Pallite VIII 120V).
Figure 8 Schematic of the split Hopkinson pressure bar system in Central South University (Not to scale)

Figure 9 Photos of the split Hopkinson pressure bar system: (a) the SHPB bars, (b) measurement systems in Central South University

4.3 High-Speed Digital Image Correlation

The DIC technique described above is implemented in Matlab to estimate in-plane surface strain fields. The procedure of the HS-DIC technique was schematically illustrated in Figure 10.
4.4 Experimental Methods

NSCB, BD, UC and passive confining tests were performed for measuring Mode-I dynamic crack initiation toughness, dynamic tensile strength, dynamic uniaxial compressive strength and dynamic triaxial compressive strength. The photographic view of dynamic testing methods was shown in Figure 11.

4.5 Dynamic Fracture Behaviour

Figure 12(a) represented a typical testing result with a striking velocity of 3.25 m/s. Ten images were captured before the stress wave arrived at the specimen calculated using Eq. 4:9 and the value indicated in Figure 12(b). It can be seen from Figure 12(c) that the time-to-fracture of SG1, $t_{1f}$, was about 40 μs and the time of crack propagating from SG1 to SG2, $t_{12}$, was approximately 15 μs.
Figure 12 Dynamic testing results of a typical NSCB test (No. FM-SCB8): (a) raw data, (b) magnitude view of the partial incident wave, (c) magnitude view of the partial data of strain gauges.

It can be seen from Figure 13, individual white belt developed from the pre-notched tips in response to loading prior to the initiation of any macroscopic observable crack, as indicated by a dashed arrow. The crack initiated at about 38 μs, after initiation, the observable crack tip formed and the crack tip position was denoted as a solid arrow. It was shown in Figure 14 that the specimen was in a state of stress equilibrium through the time-to-fracture.

Figure 13 High-speed images at different stages of the typical NSCB test (No. FM-SCB8) (The development of white belt indicated by a dashed arrow, and the observable moving crack-tip is indicated by a solid arrow in the images.)
Figure 14 Force equilibrium check of the typical NSCB test (No. FM-SCB8)

The first image (0 μs) was chosen as the reference image and a size of zone-of-interest (ZOI) of 150×95 pixels was selected for correlation calculation. The vertical strain fields were shown in Figure 15.

Figure 15 Dynamic vertical strain fields of the typical NSCB test (No. FM-SCB8) at different stages

The dynamic crack initiation toughness was approximately 1.65 GPa√m at the loading rate of 45 GPa√m/s, as shown in Figure 16. However, all SIF data to the right of the vertical becomes a function of the moving crack’s speed. This process is typically identified in the literature as dynamic crack propagation toughness that is outside the scope of this chapter. It can be seen from Figure 17 that the normalized dynamic crack initiation toughness increased almost linearly with increasing loading rates ranging from 35 to 115 GPa√m/s.
4.6 Tensile Behaviour

It can be seen from Figure 18, the time of stress equilibrium was approximately 47 μs, and the time-to-fracture was about 75 μs according to the corresponding time of the peak load.
The first image (0 µs) was chosen as the reference image and a size ZOI of 420×150 pixels was selected for correlation calculation, as shown in Figure 19. The vertical strain fields were shown in Figure 20.

It can be seen from Figure 21 that the normalized dynamic tensile strength increased with increasing loading rates ranging from 160 to 1300 GPa/s. Typical failure patterns of four rocks under different loading rates were shown in Figure 22. The main crack orientation was parallel to the impact direction and axial crack divided the specimen into at least two pieces. Two kinds of failure, i.e. shear failure and tensile failure were obviously observed. The extent of two shear failure zones at contact points of the disk depended on loading rates.
Figure 21 Normalized dynamic tensile strength of four rock types as a function of loading rate

I
II
III
IV

Figure 22 Failure patterns of Brazilian disc specimens of: (a) sandstone at loading rate of 225, 385, 620, and 786 GPa/s, (b) gabbro at loading rate of 345, 580, 842 and 1130 GPa/s, (c) coarse-grained marble at loading rate of 168, 238, 365, and 582 GPa/s, (d) fine-grained marble at loading rate of 260, 556, 830, and 1150 GPa/s,

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4.7 Uniaxial Compressive Behaviour

The first image was chosen as the reference image and a ZOI size of 185×85 pixels was selected for correlation calculation in Figure 23. The horizontal strain fields were shown in Figure 24.

Figure 23 High-speed images of the typical UC test at different stages (No. FM-UC6)

Figure 24 Dynamic horizontal strain fields of the UC test (No. FM-UC6)

The comparison of the local strains obtained using the DIC technique and the average measure of strain obtained using the 1-wave analysis method was performed. Figure 25 shows the effect of strain rate on the stress-strain curves of rock materials, i.e. the increase in compressive strength and critical strain at the maximum stress increased with
increasing strain rate, but the initial tangent modulus of rocks was not affected by strain rate. It should be noted that the slope remains linear up to higher stress level under a higher strain rate.

Figure 25 Dynamic stress-strain curves of typical UC tests (No. UC2, 4, 6 and 8)

4.8 Triaxial Compressive Behaviour

Figure 26 shows the traces of hoop strain and axial strain measured from the middle strain gauges SG2 and SG4, respectively, on different thickness of the sleeves at the striking velocity of about 25 m/s. Confining pressures of the sleeves calculated are 12.21, 16.97, 8.60, and 8.03 MPa, respectively.

Dynamic stress-strain curves were obtained by the one-wave analysis method, as shown in Figure 27. The confining pressure of the brittle-ductile transition seems to be about 8.60 MPa. Compared with the results for uniaxial compressive strength, a significant strength enhancement is observed. It can be seen that the Young’s modulus is unaffected by the strain rate with increasing strain rate and confining pressure.
Figure 27 Dynamic stress-strain curves under different confining pressures at impact velocity of 25 m/s

4.9 Summary

This section presented a detailed experimental procedure to determine dynamic crack initiation toughness, tensile strength, uniaxial compressive strength, and triaxial compressive strength in the SHPB system. The DIC technique in conjunction with high-speed photography was used to measure full-field strain fields of specimens. The proposed procedure allows the cost-effective, non-contact, full-field strain measurements of specimens in dynamic testing methods.
5 Effect of Loading Rate on Fracture Behaviour: Phenomena and Mechanisms

5.1 Experimental Procedures

The quasi-static tests were performed using a servo-hydraulic machine at the loading rate of 0.002 mm/s (Figure 28a-b). The dynamic fracture tests were carried out by means of a SHPB system. Figure 28(c) shows the photograph of loading configurations and a NSCB specimen with random speckle patterns on the surface that was applied to ensure good contrast of the images for the calculation of strain fields.

Figure 28 Experimental techniques: (a) photos NSCB specimen in the servo-hydraulic machine and high-speed photography system, (b) magnified view of (a), (c) close-up view of the partial SHPB bars and a specimen with random speckle patterns (ZOI-Zone of interest, ZOC-Zone of camera)

5.2 Dynamic Fracture Behaviour

The dynamic crack initiation toughness $K_{id}$ is the critical dynamic SIF at the time to fracture $t_f$, and the dynamic crack growth toughness $K_{id}$ is the critical SIF at a specific crack speed $V$, which are given by the following equations (Ravi-Chandar 2004)

$$K_{id}(K_{id}^{dyn}) = K_{id}^{dyn}(t_f) \quad \text{at} \quad t = t_f \quad \text{Eq. 1}$$

$$K_{id}(V;K_{id}^{dyn}) = K_{id}^{dyn}(t,V) \quad \text{for} \quad t > t_f \quad \text{Eq. 2}$$

where the dynamic loading rate is generally expressed as $K_{id}^{dyn} = K_{id}/t_f$. 


In this study, $t_f$ was measured primarily by high-speed photographs and meantime calibrated by strain gauges or crack propagation gauges. Figure 29 shows the selected sequence of high-speed photographs from dynamic fracturing tests in each rock type at a striking speed of 3 m/s. The experimental data of $K_{ia}/K_{IC}$ and $t_f$ are presented in Figure 30.

Figure 29 High-speed photographs showing dynamic fracturing process of gabbro at the striking velocity of 3 m/s (The position of observable crack tip was denoted as a solid arrow.)

Figure 30 Normalized dynamic crack initiation toughness $K_{ia}/K_{IC}$ and time to fracture $t_f$ of: (a) an analytical solution with different critical distance $\delta$ (Liu et al. 1998) and experimental data

It should be noted that in Figure 31, although the general trend of $K_{ia}/K_{IC}$ increases almost linearly with increasing normalized loading rates in the range of $K_{ic}^{high}/K_{IC} = 1\times10^4 - 4\times10^4$ s$^{-1}$, the values of $K_{ia}/K_{IC}$ in gabbro and FG marble are apparently higher than those in sandstone and CG marble, which are governed primarily by the time of stress wave required to travel through the specimen.
Figure 31 Normalized dynamic crack initiation toughness $K_{Id}/K_{IC}$ as a function of $K_{Id}^{dyn}/K_{IC}$ for selected rock materials

Figure 32(b) showed the 3D profiles of the position of crack tip and the crack opening displacement (COD) associated with the loading time for the typical dynamic test.

Figure 32 A typical NSCB test at a striking velocity of 4.8 m/s: (a) HS-images at different stages, (b) the crack tip position and crack opening displacement

As shown in Figure 33(a), at the initial stage, $v_T$ shows a decreasing trend, but $\omega$ increases with the increase of time. Figure 33(b) shows the dynamic fracture energy approaches to a low level when the crack speed $v$ is small and increases rapidly with the increase of $v$, which reveals that rock materials have the property of the crack speed-toughening. A semi-empirical rate-dependent model is proposed for the simulation of crack propagation in rock materials using $C_{sk}$. The relationship between $G_{sk}(v)$ and $v$ plotted in Figure 5:15(e) is fitted by a two-parameter expression.
Figure 33 (a) Translational and angular velocity of the flying fragments, (b) dynamic fracture energy as a function of crack speed.

Figure 34 shows the normalized $K_{ID}/K_{IC}$ increases with the increase of $v/C_R$. It is interesting to note that the rate-sensitivity of growth toughness is more evident than that of initiation toughness.

Figure 34 Normalized dynamic crack growth toughness $K_{ID}/K_{IC}$ as a function of normalized crack speed $v/C_R$.

5.3 Characterizations of Failure Mechanisms

Under quasi-static loads, there are the predominant IG microcracks with some TG contribution in FG marble (see Figure 35a-c). At the high loading rate, cleavage and noticeably rugged surfaces with a high degree of TG microcracks occurs more easily, as shown in Figure 35(d-f).

The cleavage planes perpendicular to the tensile stress have a pure normal stress across them, and others have a combination of tensile and shear stresses acting on the cleavage plane. Cracking parallel to the cleavage plane is accomplished by the lowest energy dissipation, but it requires substantially more energy as the angle increases, as shown in Figure 36.
Figure 35 SEM micrographs at increasing magnification of: (a-c) quasi-static fracture, (d-f) dynamic fracture in fine-grained marble (Micrographs were captured at a distance of ~10 mm from the notch tip.

Figure 36 Cleavages, (a) parallel to, (b) perpendicular to the cleavage plane in sandstone, (c) parallel to, (d) perpendicular to the cleavage plane in gabbro, (e) parallel to, (f) perpendicular to the cleavage plane in coarse-grained marble, (g) parallel to, (h) perpendicular to the cleavage plane in fine-grained marble
To further study the effect of loading rate on the surfaces roughness, micromeasurements were conducted using a 3D laser profilometry with a resolution of 7 \( \mu m \). Figure 37 illustrated typical 3D profiles of the fractured surfaces.

![Figure 37 3D fracture surface profiles and wireframes of: (a) a typical quasi-static specimen, (b) a dynamic specimen at crack velocity of 675 m/s (The arrow indicates the direction of crack propagation.]

5.4 Summary

Notched semi-circular bending tests were performed to study quasi-static and dynamic fracture behaviour of four well-studied rock types. On-specimen strain gauges and high-speed photography were used to determine dynamic fracture parameters at the macroscopic scale. The fracture surfaces were qualitatively and quantitatively investigated by conducting fractographic examination and roughness measurements.
6 A Micromechanical Model for Quasi-static and Dynamic Fracture

6.1 A Micromechanical Model for Determining IG/TG Fracture

When a mode I crack propagates in an elastic solid and reaches an interface, one of the following situations may occur: (a) the crack penetrates the interface and continues to propagate along its original path, i.e. crack penetration; (b) it kinks out to propagate along the interface and becomes a mixed-mode crack, which is often called ‘crack kinking/deflection’. In selected rock materials, the primary minerals, quartz and dolomite, are belong to the hexagonal crystal system (see Figure 38a). The interface can be regarded as grain boundary in the polycrystalline materials, and therefore crack kinking/deflection and crack penetration are treated as intergranular (IG) fracture and transgranular (TG) fracture, as shown in Figure 38(b). The angle between the crack plane and the grain boundary is defined as the interfacial angle, $\beta$.

The energy release rate can be obtained for various crack geometries and loading conditions, which can be expressed in terms of stress intensity factors (SIFs) $K_i$ (Anderson 2005). Therefore, the ratio of energy release rates $G_{s IG}^{IK}/G_{s TG}^{IK}$ is given by

$$
\frac{G_{s IG}^{IK}(\beta)}{G_{s TG}^{IK}} = \frac{1}{16} \left[ (3 \cos \frac{\beta}{2} + \cos \frac{3\beta}{2})^2 + (\sin \frac{\beta}{2} + \sin \frac{3\beta}{2})^2 \right]
$$

Eq. 3

The IG fracture occurs at the interface when $G_{s IG}^{IK}$ exceeds the fracture toughness of the interface (grain boundary), $K_{IC}^{IG}$; or the TG fracture occurs when $G_{s TG}^{IK}$ reaches the fracture toughness of the grain, $K_{IC}^{IG}$.

For a mixed-mode crack subjected to remote dynamic stress, in analogy with the quasi-static crack problems, the dynamic energy release rate, $G_A$, can be related to the dynamic SIFs without considering the antiplane shear mode. The ratio of two energy release rates $G_{s IG}^{IK}/G_{s TG}^{IK}$ is given by

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Figure 38 Establishment of a micromechanical model: (a) Hexagonal crystal system of the dolomite in the marble, (b) schematic diagram showing the microscopic model of intergranular and transgranular fractures, (c) formation of secondary microcracks nearly parallel to the twinning, TG, (d) formation of secondary microcracks almost normal to the twinning, TG.
\[
\frac{G_{t//}^I(\beta, v_2)}{G_d^I(v_1)} = \frac{A_t(v_1)k_t^I(v_1)(3\cos \theta + \cos \frac{3\beta}{2})^3 + A_{Ht}(v_1)k_H^I(v_1)(\sin \theta + \sin \frac{3\beta}{2})^3}{16A_t(v_1)k_t^I(v_1)}
\]

\[
= \left(\frac{v_2}{v_1}\right)^3 \left(1-\frac{v_1}{v_2}\right) \frac{R(v_1)}{R(v_2)} \frac{\alpha_0 k_t^I(v_1)3\cos \frac{3\beta}{2} + \cos \frac{3\beta}{2}^3 + \alpha_k k_t^I(v_1)(\sin \frac{3\beta}{2} + \sin \frac{3\beta}{2})^3}{16\alpha_0 k_t^I(v_1)}
\]

\text{Eq. 4}

The IG fracture kinks at the grain boundary when \( G_{d//}^I \) reaches the dynamic crack growth toughness of the interface, \( K_{t//}^I(v_2) \); or the TG fracture occurs when \( G_d^I \) reaches the dynamic crack growth toughness of the grain, \( K_{G//}^I(v_1) \).

6.2 Micro-measurement Results

It is observed from a typical fracture surface under quasi-static loads that IG fracture was the principal mode (see Figure 39a), whereas TG fracture is rarely appeared. Figure 39(b) showed a typical fracture surface of the specimens under dynamic loads at a striking speed 2.8 m/s (\( v \approx 675 \text{ m/s} \)). TG fracture was the major cracking form in the fracture surface, even though there were some IG fractures simultaneously. With the increase of impact speed or loading rate, TG fracture was becoming the dominant mode and there was little or no IG fracture, as shown in Figure 39(c).

![Figure 39 Typical SEM micrographs of fractured surfaces for the marble specimens](image)

6.3 Validation of Micromechanical Model

When the condition of \( \beta \leq 65^\circ \) is satisfied, IG fracture occurs, while TG fracture take place at \( 65^\circ < \beta < 90^\circ \). Thus, the ratio of grain-boundary fracture toughness \( K_{t//}^I \) to grain fracture toughness \( K_{G//}^I \) can be obtained as the value of 0.5. It can be seen from Figure 40 that the zone of IG fracture covered a larger range of interfacial angles (\( 0 < \beta \leq 65^\circ \)), implying that IG fracture was the dominant mode under quasi-static loads. Since the fracture toughness ratio \( K_{t//}^I / K_{G//}^I \) was 0.5 and the fracture toughness of fine-grained marble\( K_{G//}^I \) was 1.5 MPa\( \sqrt{m} \), according to the Eq. 3 and the area fractions, the values of \( K_{t//}^I \) and \( K_{G//}^I \) were obtained as 1.17 and 2.34 MPa\( \sqrt{m} \) for the grain boundary and the dolomite grain, respectively.
The ratio of normalized energy release rate \( G^\text{RI}_d (\beta, v_2) / G^\text{TG}_d (v_1) \) is plotted as a function of \( v_1 \) and \( \beta \), as shown in Figure 41. According to Eq. 4, the IG fracture occurs at \( \beta \leq 16^\circ \), and the fracture toughness ratio \( K^\text{IT}_\infty / K^G_\infty \) is obtained as the average value of 0.8. Since the measured dynamic crack growth toughness \( K^\text{IG} \) of fine-grained marble at \( v_1 = 680 \, \text{m/s} \) is about 6.1 MPa\(\sqrt{\text{m}} \), the calculated values of \( K^\text{IT}_\infty \) and \( K^G_\infty \) are approximately 5 and 6.25 MPa\(\sqrt{\text{m}} \) for the grain boundary and the dolomite grain, respectively.

As previously mentioned that \( v_2 \) is higher than \( v_1 \), setting it satisfies \( v_2 = 1.5v_1 \) and substituting Eq. 3 plot the normalized energy release rate ratios, as shown in Figure 42. It can be seen that the ratio of \( G^\text{IG}_d (\beta, v_2) / G^\text{TG}_d (v_1) \) exhibits a trend of decrease with the increase of \( v_1 \). On the basis of the criteria (Eqs. 3-4), it has also revealed that the toughness of the grains play a vital role, \( f(v_1) = [K^\text{IT}_\infty (v_2 = 1.5v_1)] / K^G_\infty (v_1) \), and the direction of crack propagation is not easily changed as the crack speed increases.
Summary

The fracture mechanics based micromechanical mode was proposed not only to explain the transition of failure modes from intergranular fracture to transgranular fracture, but also to predict the fracture toughness of the grain and the grain boundary under both quasi-static and dynamic loads. Fracture surfaces were qualitatively and quantitatively studied analysed using the SEM and an optical profilometry to identify failure mechanisms. Micro-measurements revealed that the operating failure mechanism in quasi-static tests was mostly intergranular fracture, which formed a rougher surface and resulted in a higher fractal dimension value.
7 Plate Impact Experiments and Shock Response of Rock Materials

7.1 The Plate Impact Facility

The ultra-high strain rate (UHSR) experiments were performed using the plate impact facility at the Cavendish Laboratory, as schematically shown in Figure 43. The facility consists of a projectile body and a target assembly. The barrel has a bore of 50 mm and a length of 5 m, and a projectile is launched by the gas gun at velocities up to 1,100 m/s. Figure 44 shows the images of the plate impact facility.

![Figure 43 Schematic of the plate impact facility with measurement systems (Not to scale)](image)

7.2 Measurement Techniques

The embedded manganin stress gauges gives a direct output of the stress in the material of interest. Stress gauges placed in two distinct planes are used to determine longitudinal and lateral stresses interrogated by the piezoresistive manganin gauges, respectively, as shown in Figure 45.
In the present study, the photon Doppler velocimetry (PDV) system is used to measure the free surface velocity of the target, as shown in Figure 46. The systems use Dopplet-shifted light reflected diffusely or specularly to infer velocity data, and can be used to detect free surface motion or to detect interfacial motion between the desired sample and an optically transparent window.

7.3 Testing Methods

Plate impact experiments are conducted to determine Hugoniot curves, spall tensile strength, shear strength and Hugoniot elastic limit (HEL) of the fine-grained marble and coarse-grained gabbro. Two different types of shock compression tests (i.e., reverse and conventional impact) were performed respectively, as shown in Figure 47(a-b). Two distinct experimental setups to determine spalling tensile strength are employed, as shown in Figure 47(c-d). Figure 47(e) shows a typical plate impact target where two lateral gauges were embedded.
Figure 47 (a) Schematic of the reverse impact experiment, (b) schematic illustration (exploded) of a target enclosed within two longitudinal gauge packages and two rock plates, (c-d) schematics of spalling experimental setups without and with PMMA windows, and (e) schematic illustration (exploded) of a rock target containing one longitudinal gauge and two embedded lateral gauges.
7.4 Experimental Results and Discussion

The Hugoniot data derived from the gauge and HetV records are shown in Figure 48. It can be seen that the Hugoniot in pace of the longitudinal stress and particle velocity are well-fitted by a linear fit for gabbro $\sigma_x = 7.51 u_p + 7.92 u_p^2$, and by a second order polynomial fit for marble, $\sigma_x = 7.51 u_p + 7.92 u_p^2$, respectively.

![Figure 48 Measured Hugoniot states in stress particle velocity space for: (a) marble, (b) gabbro](image)

It is possible therefore to plot this against the gradients of straight line fits to Hugoniot data, as shown in Figures 49-50.

![Figure 49 Plot of impedance, $Z_s$, against Hugoniot, $Z_S$, slope for various literature data](image)

![Figure 50 Equation of state (EOS) of (a) igneous rocks and (b) metamorphic rocks](image)
7.5 Summary

Plate impact experiments are conducted to determine Hugoniot curves, spall tensile strength, shear strength and Hugoniot elastic limit of the marble and gabbro. Manganin stress gauges and a laser interferometer HetV system are used for the measurement of stress and shock particle velocity, respectively. The Hugoniots of two rocks were able to be described by a linear relationship in pressure particle-velocity space, which were also similar with the theoretical elastic behaviour calculated from the elastic impedance. Attempts were made to investigate the dynamic tensile properties. Lateral stresses were measured using embedded lateral gauges. The shear strength has been calculated during plate impact in the axial stress range 3-12 GPa, and is shown to increase significantly with shock stress.
8. Principal Findings and Conclusions

8.1 A Critical Review of Dynamic Experimental Techniques and Mechanical Behaviour

The loading techniques commonly used for intermediate strain rate testing of rock materials are pneumatic-hydraulic, completely gas-driven, and drop-weight machines. At high strain rate, the split Hopkinson bar has been widely used, and major developments of this technique for rock materials are summarized in detail. The plate impact techniques have been successfully employed to determine shock properties at the ultra-high strain rate.

In terms of measurement techniques, a detailed description is made to present the principles and applications of the most frequently used optical methods, including photoelastic coating, moiré, caustic, holographic interferometry, digital image correlation, and infrared thermography. The testing methods to obtain dynamic strength and fracture roughness as well as other parameters are primarily extended or modified from quasi-static methods.

Dynamic uniaxial and triaxial compressive, tensile and shear strength and fracture toughness are obtained by quantitative assessment of corresponding testing methods, and the influencing factors and failure patterns are also discussed. Dynamic mechanical properties and fracture behaviour change with the loading rate; in particular, the responses distinguishably change after the strain/loading rate exceeds a critical value.

The effects of some influencing environmental factors and intrinsic rock factors are presented. Several popular semi-empirical rate-dependent equations are put forward for predicting dynamic strength. On the basis of experimental data and the physical mechanisms of strain-rate effect, phenomenological and physically based rate-dependent constitutive models for modelling mechanical behaviour over a wide range of loading rates. The energy- and stress-based dynamic fracture criteria are summarized, among which the dynamic Mohr-Coulomb and stress intensity factor based criteria have been widely used for rock-like materials.

8.2 Quantitative Determination of Dynamic Mechanical Properties

Four types of testing methods, namely notched semi-circular bending, Brazilian disc, uniaxial compression, and passive confining pressure tests are conducted using a modified SHPB technique. Detailed experimental procedures and calibrations are presented to determine dynamic Mode-I crack initiation and propagation toughness, tensile strength, uniaxial compressive strength, and triaxial compressive strength. Moreover, the DIC technique in conjunction with high-speed photography is developed to record fracturing process and to measure full-field deformation fields. Hugoniot elastic limit, spall tensile strength and shear strength of rock materials under shock loading are obtained using Cambridge plate impact technique. Longitudinal and lateral stresses are measured by manganin gauges embedded in specimens. A HetV system is applied to measure the rear surface velocity. The methods used to make measurements are, to great extent, standard experimental procedures when determining dynamic mechanical properties.

8.3 High-speed Optical Measurement Techniques

The DIC in conjunction with high-speed photography is developed for the measurement of dynamic displacement/strain fields. A close match between strain gauges and DIC measurements has shown that the HS-DIC technique can reliably be used to measure full-field surface characteristics under dynamic loads. Especially, the DIC technique continues strain measurements after failure of on-specimen contact measurement techniques. A HetV system is successfully used for measuring free surface velocities of inhomogeneous rock-like materials.

8.4 Micromechanical Modelling of Failure Mechanisms

The SEM fractographic studies reveal that the operating failure mechanisms under dynamic loads are mostly of transgranular microcracks. With the increase of loading rate, the number of some typical microcrack modes, such as multiple cleavage steps, opening microcracks and microbranches, is increased rapidly. The increases of surface areas generated by these typical modes are responsible for large energy dissipation, while it cannot be identified well by the surface roughness measurements.
The 3D surface roughness measurements show that the fractal dimension of rock material is dependent primarily on microcrack modes. The effect of loading rate on the fractal dimension is unapparent in sandstone, perhaps due to the complicated cement fracture and surface pits.

Based on quasi-static and elasto-dynamic fracture mechanics and macroscopic experimental results, a micromechanical mode is proposed not only to explain the failure transitions from intergranular fracture to transgranular fracture, but also to predict the fracture toughness of the grain and the grain boundary under both quasi-static and dynamic loads.
9. Selected References


