Understanding Rock Stress

John A Hudson
Rock masses already contain rock stresses.

Do we understand rock stress? Why do we need to understand it?

And can it be reliably determined at a particular site?

Here are the fundamentals…hopefully presented in a relatively simple way.

It’s essential to understand stress to work in rock mechanics and rock engineering.
The basic motivations for understanding *in situ* rock stress are two-fold.

1. To have a basic understanding and knowledge of the stress state for interpreting structural geology features and for engineering analyses. How did the rock become deformed and/or fractured in that way, and for engineering what stress effects are we defending ourselves and our structures against?

2. To have a specific and 'formal' knowledge of the boundary conditions for stress analyses conducted in the design phase of rock engineering projects.

There are many cases in rock engineering where the stresses are not applied as such; rather, the stress state is altered by the engineering activities, e.g. in the case of excavating a rock slope or tunnel.

The rock is currently pre-loaded, i.e. pre-stressed
The generic rock mechanics/rock engineering problem

So, let’s think about rock stress in the context of our generic diagram.
Characterising the forces on and throughout the rock mass through the concept of stress

What is the stress at a point within a rock mass?
Point 1: The need to understand rock stress

Stresses in rock masses have caused the geological structures that we see today and knowledge of these is required for the modelling and design of current engineering projects.

During rock engineering, the pre-existing natural rock stress state can be significantly altered.

Stress is not a familiar concept.
**Point 2: Stress is not a scalar or vector quantity, it is a tensor quantity**

A **scalar** quantity has magnitude only, e.g. temperature, energy (one component required for its specification, e.g. 6°C).

A **vector** quantity has magnitude and direction, e.g. velocity, force (three components required for its specification, e.g. 6 N acting along a line with trend 234° and plunge 6°).

A **tensor** quantity has magnitude, direction and the plane that it is acting on, e.g. stress and strain (six components required for its specification).
A solid can sustain a shear force, whereas a liquid or gas cannot.

A liquid or gas contains a pressure, i.e. a force per unit area, which acts equally in all directions and hence is a scalar quantity.

Rock can sustain a shear force and hence contains stress, which is a tensor quantity.
Point 3: There are normal forces and there are shear forces, and there are normal stress components and shear stress components.

The force can be resolved into a normal component, $F_n$, acting perpendicular to the plane, and a shear component, $F_s$, acting parallel to the plane.

A normal stress, $\sigma$, can be resolved into a normal stress component, $\sigma_n$, and a shear stress component, $\tau$. However, while the resolved force perpendicular to the plane $F_n = F \cos \theta$, the resolved stress perpendicular to the plane, $\sigma_n = \sigma \cos^2 \theta$ because the stress quantity is related to force/area: while $\cos \theta$ is sufficient for the force resolution, $\cos^2 \theta$ is required for the stress component resolution—because both the force and the area have to be resolved in the case of stress.
**Point 4: Stress is a point property**

Considering a small cube within a rock, the normal and shear stress components are as indicated with the notation explained at the right-hand side of the Figure: with reference to the axes directions shown on the left, the first subscript for a stress component indicates the plane on which the stress component is acting; the second subscript indicates the direction in which the stress component is acting.

These stress components are considered as limits when the volume of the cube is reduced to zero so that stress is regarded as a point property.
**Point 5: The nine stress components can be listed out in matrix form**

The nine stress components can be listed out in a table form. This is known as the stress tensor. It is a more complex quantity than a scalar or vector because the resolution of all the components when the cube is rotated is more complicated than simply resolving forces.
**Point 6: The stress matrix is symmetrical.**

Some of the off-diagonal terms in the matrix are equal. This is a result of ensuring that the cube does not rotate. In the x-y plane shown above, the cube will not rotate if \( \tau_{xy}\Delta l = \tau_{yx}\Delta l \), in other words if \( \tau_{xy} = \tau_{yx} \), and similarly for the x-z and y-z planes, \( \tau_{xz} = \tau_{zx} \) and \( \tau_{yz} = \tau_{zy} \).
**Point 7: The state of stress at a point has six independent components.**

As a result of the equality of shear stress components in each plane, only six of the nine components in the stress matrix are independent.

The consequence is that the state of stress at a point has six independent components with the corollary that the stress at a point in a rock mass can only be specified by designating the six components.

These can be the six components in the matrix given with reference to a set of axes or, more usually, they are given with reference to the magnitude and orientations of the principal stresses (see the next Point).

If less than six components are given, the statement has no meaning, e.g. the stress is 6 MPa has no meaning because six components are required to specify the stress state.
**Point 8:** There is an orientation in space for which all the shear stresses vanish and there are only normal components of stress—the principal stresses.

It is perhaps surprising that, given the existence of the six shear stress components, it is always possible to find an orientation of the cube for which all the shear stresses disappear leaving only normal stresses acting.

These normal stresses are known as the principal stresses, the major, intermediate and minor principal stresses: $\sigma_1$, $\sigma_2$ and $\sigma_3$ respectively.

The converse is not true: we cannot find an orientation of the cube such that all the normal stresses disappear leaving only principal shear stresses.
Stress analysis can become complicated, and is beyond the scope of these lectures...

Three dimensional transformation of tensors

When stress components are known in an \(xyz\) coordinate system and are required in a another coordinate system \(lmn\) inclined with respect to the first, then the matrix equation to conduct stress transformation is

\[
\begin{bmatrix}
\sigma_l & \tau_{lm} & \tau_{ln} \\
\tau_{ml} & \sigma_m & \tau_{mn} \\
\tau_{nl} & \tau_{mn} & \sigma_n
\end{bmatrix}
= \begin{bmatrix}
l_x & l_y & l_z \\
m_x & m_y & m_z \\
n_x & n_y & n_z
\end{bmatrix}
\begin{bmatrix}
\sigma_x & \tau_{xy} & \tau_{xz} \\
\tau_{yx} & \sigma_y & \tau_{yz} \\
\tau_{zx} & \tau_{zy} & \sigma_z
\end{bmatrix}
\begin{bmatrix}
l_x & m_x & n_x \\
l_y & m_y & n_y \\
l_z & m_z & n_z
\end{bmatrix}
\]
or

\[
\sigma_{lmn} = R \cdot \sigma_{xyz} \cdot R^T,
\]

where the matrix \(R\) is known as the rotation matrix. The term \(l_x\) is called the direction cosine of the angle between the \(x\)-axis and the \(l\)-axis, and physically it is the projection onto the \(x\)-axis of a unit vector parallel to the \(l\)-axis. The other terms are similarly defined.

Matrix multiplication gives the normal stress component in the \(l\) direction as:

\[
\sigma_l = l_x^2 \sigma_x + l_y^2 \sigma_y + l_z^2 \sigma_z + 2(l_x l_y \tau_{xy} + l_x l_z \tau_{xz} + l_y l_z \tau_{yz})
\]

and the shear stress component on the \(l\) face in the \(m\) direction as:

\[
\tau_{lm} = l_x m_x \sigma_x + l_y m_y \sigma_y + l_z m_z \sigma_z + (l_x m_y + l_y m_x) \tau_{xy} + (l_x m_z + l_z m_x) \tau_{xz} + (l_y m_z + l_z m_y) \tau_{yz}.
\]

The four remaining equations are found using cyclic permutation (i.e. \(x \rightarrow y\), \(y \rightarrow z\), \(z \rightarrow x\), \(l \rightarrow m\), \(m \rightarrow n\), \(n \rightarrow l\)) of the subscripts in these equations.

Thus, if we know the orientation of each of the axes in the \(xyz\) and \(lmn\) coordinate systems, together with the stress tensor in the \(xyz\) coordinate system, we can calculate the transformed stress tensor.

On many occasions it is convenient to refer to the orientation of a plane on which the components of stress are required to be known using the dip direction/dip angle \((\alpha, \beta)\) notation. The dip direction is always given as a clockwise bearing from North and the dip is quoted as an angle between 0° and 90° measured downwards from the horizontal plane. If we employ a right handed coordinate system with \(x = North\), \(y = East\), and \(z = Down\) (the reason for choosing this system is to simplify the calculation when using compass bearings), then the direction cosines of the \(l\)-axis are computed as

\[
l_x = \cos \alpha \cos \beta; \quad l_y = \sin \alpha \cos \beta; \quad l_z = \sin \beta.
\]

Hence, the rotation matrix is

\[
\begin{bmatrix}
l_x & l_y & l_z \\
m_x & m_y & m_z \\
n_x & n_y & n_z
\end{bmatrix}
= \begin{bmatrix}
\cos \alpha \cos \beta & \sin \alpha \cos \beta & \sin \beta \\
\cos \alpha \sin \beta & \sin \alpha \sin \beta & \cos \beta \\
\cos \alpha & \sin \alpha & 0
\end{bmatrix}
\begin{bmatrix}
l_x & l_y & l_z \\
m_x & m_y & m_z \\
n_x & n_y & n_z
\end{bmatrix}
\]

Now, if we take the line of maximum dip to be the \(n\)-axis and the \(l\)-axis to be a horizontal line on the plane, then we have

\[
\alpha_l = 90 + \alpha_n, \quad \beta_l = 0
\]

\[
\alpha_n = 180 + \alpha_n, \quad \beta_n = 90 - \beta_n
\]

providing we ensure that the dip direction values always remain between 0° and 360°. The rotation matrix then becomes

\[
\begin{bmatrix}
l_x & l_y & l_z \\
m_x & m_y & m_z \\
n_x & n_y & n_z
\end{bmatrix}
= \begin{bmatrix}
-\sin \alpha_n & \cos \alpha_n & 0 \\
-\cos \alpha_n \sin \beta_n & -\sin \alpha_n \sin \beta_n & \cos \beta_n \\
\cos \alpha_n \cos \beta_n & \sin \alpha_n \cos \beta_n & \sin \beta_n
\end{bmatrix}
\begin{bmatrix}
l_x & l_y & l_z \\
m_x & m_y & m_z \\
n_x & n_y & n_z
\end{bmatrix}
\]

We often have to solve the inverse problem, i.e. we know the stress state in the \(lmn\) system and wish to calculate the stress state in the \(xyz\) system. To do this, we simply take the first equation and rearrange it, using the properties of matrices, to find

\[
\begin{bmatrix}
\sigma_x & \tau_{xy} & \tau_{xz} \\
\tau_{yx} & \sigma_y & \tau_{yz} \\
\tau_{zx} & \tau_{zy} & \sigma_z
\end{bmatrix}
= \begin{bmatrix}
l_x & m_x & n_x \\
l_y & m_y & n_y \\
l_z & m_z & n_z
\end{bmatrix}
\begin{bmatrix}
\sigma_l & \tau_{lm} & \tau_{ln} \\
\tau_{ml} & \sigma_m & \tau_{mn} \\
\tau_{nl} & \tau_{mn} & \sigma_n
\end{bmatrix}
\begin{bmatrix}
l_x & l_y & l_z \\
m_x & m_y & m_z \\
n_x & n_y & n_z
\end{bmatrix}
\]
or

\[
\sigma_{xyz} = R^T \cdot \sigma_{lmn} \cdot R.
\]
**Point 9: All free surfaces are principal stress planes.**

Whilst there can be shear stresses within a solid, there can be no shear stresses on a free surface—because there can be no reaction to sustain the shear stress.

It follows immediately that a free surface is then a principal stress plane, i.e. the small cube must be orientated with one face being the free surface.

Moreover, there is also no normal stress acting on a free surface and so the associated principal stress acting perpendicular to the surface is zero.
This is for the case where there are no tensile stresses in the rock, i.e. the $\sigma_3$ value of zero is the lowest of the three principal stresses.
Influence of an open fracture on the local stress state
Plotting the stress state

The diagram indicates a stress state

\[ \sigma_1 = 16 \text{ MPa} \text{ acting horizontally north-south}, \]

\[ \sigma_2 = 10 \text{ MPa} \text{ acting vertically, and} \]

\[ \sigma_3 = 7 \text{ MPa} \text{ acting horizontally east-west.} \]
A hemispherical projection allows evolution of orientation to be shown, but not magnitude. However, at any given stage of the stress path the principal stresses can be seen to be orthogonal.

Complete visualisation requires a composite diagram. Here, evolution of the stress path is again shown by the letters a through to e, but now the orientation and the magnitude are shown in terms of position on the hemispherical projection and height above it.

Diagrams thanks to Prof John Harrison.
Point 10: Measuring rock stress is not an easy task

This is due to a combination of adverse factors, including the facts that

• stress is a point property,

• the stress state must be altered in order for the measurements to be made,

• at least six measurements are required,

• local measurements may not indicate the overall stress state, and

• the stress measurements are often made remotely in boreholes.
**Natural stress:** the *in situ* stress which exists prior to engineering.

**Induced stress:** the natural stress state as perturbed by engineering.

**Gravitational stress:** the stress state caused by the weight of the rock above.

**Tectonic stress:** the stress state caused by tectonic plate movement.

**Residual stress:** the stress state caused by previous tectonic activity.

**Thermal stress:** the stress state caused by temperature change.

**Palaeostress:** a previous natural stress that is no longer acting.

**Near-field stress:** the stress state in the region of an engineering perturbation.

**Far-field stress:** the stress state beyond the near-field.

**Local stress:** the stress state in a region of interest.
Different scales

- Tectonic scale and regional stresses
- Site scale
- Excavation scale
- Borehole/measurement scale
- Microscopic scale
Stress Perturbation Factors

- Rock inhomogeneity
- Rock anisotropy
- Rock discontinuities
- Influence of a free surface

\[
\begin{bmatrix}
\sigma_1 & 0 & 0 \\
0 & \sigma_2 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

Principal stresses parallel to excavation surface

Principal stress perpendicular to excavation surface

$\sigma_3 = 0$
Influence of a fracture zone on the local rock stress: AECL URL, Manitoba

Diagram thanks to Prof Derek Martin
From “Geological Engineering” by Gonzalez de Vallejo & Ferrer
Case Example of *In Situ* Stress

(next four slides from **Max Lee** - AMC Consultants Pty Ltd, Australia)
Principal Stress Magnitudes versus Depth

Yilgarn Craton

Note the spread of results in each case.

Eastern Australia
Note how the stress trends have been clarified by plotting \( I = \sigma_1 + \sigma_2 + \sigma_3 \) against depth.
Principal Stress Magnitudes vs 1st Stress Invariant: Combined Data

Yilgarn + Eastern Australian data

- Same relationship!!
- Two hypotheses
  - Structures control the principal stress ratios that rock masses can sustain, irrespective of how strong or weak the intact rock might be
  - It’s likely that all rock masses try to sustain the maximum “geologic” stress that they can sustain

…and that the same relationship emerges for West and East Australia.
Your lecturer in Japan with three world experts on rock stress and its measurement!
Issue of this journal devoted to rock stress estimation which contains the four International Society for Rock Mechanics Suggested Methods and 17 papers on experiences of stress measurement.
ISRM Suggested Methods for Rock Stress Estimation

Part 1: Strategy for rock stress estimation
Part 2: Overcoring methods
Part 3: Hydraulic fracturing and/or hydraulic testing of pre-existing fractures (HTPF) methods
Part 4: Quality control of rock stress estimation

The Suggested Methods have been published in a Rock Stress Estimation Special Double Issue of the International Journal of Rock Mechanics and Mining Sciences (Volume 40, Issue 7-8, 2003), together with a suite of supporting papers on various aspects of establishing the rock stress state.
End of Lecture 2