Rock and Rock Mass Deformability (Compressibility, Stiffness…)  

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Common Symbols in RM

- **E, ν**: Young’s modulus, Poisson’s ratio
- **φ**: Porosity (e.g. 0.25, or 25%)
- **c′, φ′, T₀**: Cohesion, friction \( \angle \), tensile strength
- **T, p, p₀**: Temperature, pressure, initial pres.
- **σ_v, σ_h**: Vertical and horizontal stress
- **σ_{hmin}, σ_{HMAX}**: Smallest, largest horizontal \( σ \)
- **σ_1, σ_2, σ_3**: Major, intermediate, minor stress
- **ρ, γ**: Density, unit weight \( (γ = ρ × g) \)
- **K, C**: Bulk modulus, compressibility

*These are the most common symbols we use*
Obtaining Rock Deformability

Properties data bank

<table>
<thead>
<tr>
<th>Depth</th>
<th>Fric.</th>
<th>Coh.</th>
<th>XXX</th>
<th>YYY</th>
<th>ZZZ</th>
</tr>
</thead>
</table>

Commercial, in-house data banks

Rock Properties

(E, ν, φ, c', C, k, ...)

Core data

1-D Measuring Rock Properties

Borehole seismics

3-D Seisms

Reflected and direct paths

Log data

REG. TIPO

SVS-337
Stress and Pressure

- Petroleum geomechanics deals with stress & pressure
- Effective stress: “solid stress”
- Pressure is in the fluid phase
- To assess the effects of $\Delta \sigma'$, $\Delta p$, $\Delta T$, $\Delta C$…
- **Rock properties** are needed
  - Deformation properties…
  - Fluid transport properties…
  - Thermal properties…

\[ \sigma_a - \text{axial stress} \]
\[ \sigma_r - \text{radial stress} \]
\[ p_o - \text{pore pressure} \]
\[ \sigma_a = \frac{F_a}{A} \]
To solve a $\sigma'$-$\varepsilon$ problem, we must know how the rock deforms (strain - $\varepsilon$) in response to $\Delta\sigma'$.

This is often referred to as the “stiffness” (or compliance, or elasticity, or compressibility…).

For “linear elastic” rock, only two parameters are needed: Young’s modulus, $E$, and Poisson’s ratio, $\nu$ (see example, next slide).

For more complicated cases - plasticity, dilation, anisotropic rock, salt, etc. - more and different parameters are needed.
The Linear Elastic Model

- The stiffness is assumed to be constant (E)
- When loads are removed, deformation are reversed
- Suitable for metals, low φ rocks such as...
  - Anhydrite, carbonates, granite, cemented sands...
- For many petroleum geomechanics problems, linear elastic assumptions are sufficient

\[
\sigma_1 = \sigma'_a
\]

\[
\sigma_3
\]

\[
\text{Stress} - \Delta \sigma'_a = (\sigma_1 - \sigma_3)
\]

\[
\text{Strain} - \varepsilon_a
\]

\[
E_1 \text{ is stiffer than } E_2
\]
Definitions of $E$ and $\nu$?

Young’s modulus ($E$): $E$ is how much a material compresses under a uniaxial change in effective stress - $\Delta \sigma'$

$$E = \frac{\Delta \sigma'}{\varepsilon}$$

Poisson’s ratio ($\nu$): $\nu$ is how much rock expands laterally when compressed. If $\nu = 0$, no expansion (e.g.: a sponge).
- For sandstones, $\nu \approx 0.2-0.3$
- For shales, $\nu \approx 0.3-0.4$

$$\nu = \frac{\Delta r}{\Delta L}$$

Triaxial Test

deformation

radial dilation $\Delta r$

strain ($\varepsilon$) = $\frac{\Delta L}{L}$

$L$

$\Delta L$
1D and 3D Compressibility?

- Change in volume with a change in stress
- In 1-D compressibility, lateral strain, $\varepsilon_h$, = 0
  - Often used for flat-strata compaction analysis
- 3-D compressibility involves all-around $\Delta \sigma'$
- $C_{3D} = 1/K$, where $K$ is the bulk modulus of elasticity
Some Guidelines for Testing...

- Use high quality core, as “undisturbed” as possible, under the circumstances...
  - Avoid freezing, other severe treatments
  - Preserve RM specimens on the rig floor if you can
- Use as large a specimen as possible
  - A large specimen is more representative
  - Avoid “plugging” if possible (more disturbance)
- If undisturbed core is unavailable
  - Analogues may be used
  - Data banks can be queried
  - Disturbed samples may be tested with “judgment”
More Guidelines for Testing…

- Replicating *in situ* conditions of $T$, $p$, $[\sigma]$ is “best practice” (but not always necessary)
- Following the stress path that the rock experiences during exploitation is “best practice”
- Test “representative” specimens of the GMU
- Testing jointed rock masses in the laboratory is not feasible; only the “matrix” of the blocks…
- It is best to combine laboratory test data with log data, seismic data, geological models, and update the data base as new data arrive…
### Laboratory Stiffness of Rocks

- From cores, other samples: however, these may be microfissured ($E_{\text{field}}$ may be underestimated)
- In microfissured or porous rock, crack closure, slip, contact deformation may dominate stiffness
- $E_S$ and $v_S$ (static) under $\sigma'_3$ gives best values
- If joints are common *in situ*, they may dominate rock response, but are hard to test in the lab

10 m ~ 0.10 m
Typical Test Configuration…

- An “undamaged”, homogeneous rock interval is selected
- A cylinder is prepared with flat parallel ends
- The cylinder is jacketed
- Confining stress & pore pressure are applied
- The axial stress is increased gradually
- $\varepsilon_a$, $\varepsilon_r$ measured

\[ \sigma_a, \sigma_r, \varepsilon_r \]
Strain gauges measure strains (or other special devices can be used)

Pore pressure can be controlled, and...

$\Delta V_{\text{pore}}$ can be measured at constant backpressure

Similar set-up for high-T tests and creep tests

Acoustic wave end caps

Etc…
A Simpler Standard Triaxial Cell

- Developed by Evert Hoek & John Franklin
- Is a good basic cell for rock testing
- Standard test methods are published by the International Society for Rock Mechanics (ISRM)
In the Laboratory...

- Axial deformation is measured directly by the movement of the test platens
- Bonded strain gauges on the specimen sides are also used
  - Gives axial strain (calculate $E$)
  - Also gives the lateral strain (calculate $v$)
- Special methods for porous rocks or shales because strain gauges don’t work well
- High T tests, acoustic velocity measurements during tests, etc., etc.
Reminder: Heterogeneity!

These materials respond radically different to stress: one flows, the other fractures. How might we incorporate such behavior in our testing and modeling for a natural gas storage cavern?
Reminder: Scale and Heterogeneity
Reminder: Anisotropy

- Different directional stiffness is common!
  - Bedding planes
  - Oriented minerals (clays usually)
  - Oriented microcracks, joints, fissures…
  - Close alternation of thin beds of different inherent stiffness (laminated or schistose)
  - Imbricated grains
  - Different stresses = anisotropic response
  - Anisotropic grain contact fabric, etc.
Reminder: Anisotropy

\[ M = \frac{\Delta \sigma'}{\frac{\Delta L}{L}} \]

Vertical core

Bedding inclination

Apparent axial stiffness - \( M \)

\( \Delta \sigma' \)

\( \theta \)

0° 30° 60° 90°

0° 30° 60° 90°

e.g.: shales, laminated strata
Orthotropic Stiffness Model

- In some cases, it is best to use an orthotropic stiffness model - shale
- Vertical stiffness and Poisson’s ratio are different than the horizontal ones
- Properties in the horizontal plane the same
- This is as complex as we want: $E_1, E_2, \nu$

Layered or laminated sedimentary strata

Strata subjected to different stresses

Shales (clay minerals)

Orthotropic elastic model
Lab Data, Then What?

- Clearly, laboratory tests are valuable, but insufficient for design and optimization…
- We also use correlations from geophysical logs
  - Obtain relevant, high-quality log data
  - Calibrate using lab test data
  - Use logs and 3-D seismic to extrapolate and interpolate (generating a 3-D whole earth model)
Reminder: Scale Issues...

Rock vs Rock mass

- Intact rock
- Single discontinuities
- Two discontinuities
- Several disc.
- Rockmass

Laboratory specimen ("intact")

A tunnel in a rock mass

70-200 mm

20-30 m
Rock Mass Stiffness Determination

- Use correlations based on geology, density, porosity, lithology…
- Use seismic velocities \((v_P, v_S)\) for an upper-bound limit (invariably an overestimate)
- Measurements on specimens in the lab? (problems of scale and joints)
- *In situ* measurements
- Back-analysis using monitoring data such as compaction measurements…
- Reservoir response to earth tides…
**In Situ Stiffness Measurements**

- Pressurization of a packer-isolated zone, with measurement of radial deformation ($\Delta r/\Delta \sigma'$), in an “impermeable material” so that $\Delta \sigma' = \Delta p_w$
- Direct borehole jack methods (mining only)
- Geotechnical pressure-meter modified for high pressures (membrane inflated at high pressure, radial deformation measured)
- Hydrofracture flexing (THE™ tool, rarely used and quite expensive)
- Correlations (penetration, indentation, others?)
Seismic Wave Stiffness ($E_D$, $\nu_D$)

- $v_p$, $v_s$: dynamic responses are affected by stress, density and elastic properties ($\sigma$, $\rho$, $E$, $\nu$)
- Seismic strains are tiny ($<10^{-8}$-$10^{-7}$), they do not compress microcracks, pores, or contacts
- Thus, $E_D$ is **always higher** than the static test moduli, $E_S$
- The more microfissures, pores, point contacts, the more $E_D > E_S$, x 1.3 to x 10 (for UCSS)
- If porosity $\sim 0$, $\sigma'$ very high, $E_S$ approaches $E_D$
- Seismic moduli should be calibrated by testing
Seismic (Dynamic) Parameters

△t is transit time, plotted in microseconds per foot or per metre

V_p and V_s are calculated from the transit time and the distance – L – between the receiver and the transmitter in the acoustic sonde

V_p = L/△t_p
V_s = L/△t_s

Dynamic Elastic Parameters:

v_D = \[\frac{V_p^2 - 2V_s^2}{2(V_p^2 - V_s^2)}\]

E_D = \[\frac{\rho_b \cdot V_s^2(3V_p^2 - 4V_s^2)}{(V_p^2 - V_s^2)}\]

μ_D = ρ_b \cdot V_s^2 (shear modulus)
Deformation Properties from Logs

- Simple P-wave transit time correlations
- Dipole sonic data - $V_p$ and $V_s$
  - Often dipole sonic data not available
  - Estimate $V_s$ from $V_p$, ratios, lithology...
- Basic data necessary for estimation:
  - Sonic log, preferably dipole
  - Density log (gamma-gamma)
  - Water saturation log (for corrections)
  - Mineralogy/lithology logs (for corrections)
- Service companies provide these methods

Use with JUDGMENT!
Damage will alter the sonic velocities.

Wave trains

Velocity curve is offset because of lower $v$ near the borehole wall (damage)

Attenuation per metre can also be used to relate to damage

Arrival time delay from damage
Back-Analysis for Stiffness

- Apply a known effective stress change, measure deformations (e.g., uplift, compaction)
- Use a mathematical model to back-calculate the rock properties (best-fit approach)
- Includes all large-scale effects
- Can be confounded by heterogeneity, anisotropy, poor choice of GMU, ...
- Often used as a check of assumptions
- One must commit to some monitoring (e.g., $\{\Delta z\}$) in order to achieve such results
Discontinuities & E

- Grain contact deformability is responsible for sandstone stiffness.
- These may be cemented or not, and in low-$\phi$ media, they become interlocked, rocks are stiffer.

\[ f_n = \text{normal force} \]
Cracks and Grain Contacts

Microflaws can close or open as $\sigma'$ changes

Flaws govern rock stiffness

The contact fabric and $\phi$ dominate the stiffness of porous SS

Fissures are more important in limestones, as well as $\phi$
A grain contact solution was developed 120 years ago by Hertz:
\[ \Delta d \propto \frac{1}{E}, \quad (\Delta F)^{\frac{2}{3}} \]
It shows that grain-to-grain contacts become stiffer with higher load.
High \( \phi \) rocks dominated by such contacts.
They are stiffer with stress:
\[ C = f(\sigma') \]
Non-Linear Elastic Behavior

- The stiffness is assumed to be variable – $E(\sigma'_3)$
- Deformation is still reversible
- Suitable for highly microfissured materials, high $\phi$ granular rocks
- For some geomechanics problems, a non-linear elastic solution is useful
  - Sand compaction, sand production…

\[ \sigma_1 - \sigma_3 = E_1 \]
\[ E_2 = f(\sigma') \]

Stress - ($\sigma_1 - \sigma_3$) vs Strain - $\varepsilon_a$
Real Rock Behavior

- Unconsolidated sandstones have a stiffness that is a function of effective stress \( \sigma' \):

**Empirical relationships**

\[
\begin{align*}
E &= a + b\sigma' \\
E &= a + b\sigma^c
\end{align*}
\]

Linear elastic (\( E = \text{constant} \))
Geological Factors and Stiffness

- Geological history can help us infer the stiffness and the response to loading…
- Intense diagenesis
  - Reduces porosity
  - Cementation
- Deeper burial then erosion (precompaction)
- Age (in general correlated to stiffness)
- Porosity (lower $\phi$, higher $E$)
- Mineralogy ($\text{SiO}_2$ vs. litharenite mineralogy)
- Tectonic loading (reduces $\phi$…)
Sandstone Stiffness & Diagenesis

\[ \sigma_{ij} \]

\[ \Delta l = \Delta \phi \]

\[ T, t, \sigma', p \]

chemistry

high \( \sigma \), small \( A \)

low \( \sigma \), large \( A \)

DIAGENESIS!
Precompaction Effect by Erosion

The sand is far stiffer than expected because of precompaction! - e.g. Athabasca Oil Sand
High-Porosity Chalk

- Hollow, weak grains (coccoliths)
- Weak cementation (dog-tooth calcite)
- Weak, cleavable grain mineral (CaCO$_3$)
North Sea Chalk, high $\phi$ Coal, Diatomite, are quasi-stable, collapsing rocks at some $\sigma'_v$. $\log(\sigma'_v)$ vs. porosity graph showing normal densification, cementation effect, initial stiff response, apparent threshold $\Delta\sigma'$, and collapse when grain cement ruptured.

"virgin" compression curve
Sandstone Diagenesis

- Dense grain packing
- Many long contacts
- Concavo-convex grain contacts
- $\text{SiO}_2$ precipitated in interstitial regions
- Only 1% solution at contacts = 8% loss in volume
- A stable interpenetrative fabric, high stiffness

Fine-grained unconsolidated sandstone
Effect of Diagenetic Densification

- diagenetic porosity loss @ constant $\sigma'$
- present state
- apparent threshold $\Delta\sigma'$
- "virgin" compression curve

$\log(\sigma'_v)$

porosity

stiff load response
“Precompaction” Effect

- A threshold value is necessary before any non-elastic compression is triggered
- This may arise from three processes
  - True precompaction by burial then erosion
  - Cementation of grains = stiffer + stronger
  - Prolonged diagenesis increases stiffness
- Little deformation is seen in early drawdown, but occurs later
- This can confuse field planning
Issues to Remember...

- Stiffness (elastic modulus) is a fundamentally important rock property for analysis
- We can measure it with cored rock specimens
- Also, in boreholes (much more rarely)
- Sometimes, through correlations to other measures such as geophysical data
- Sometimes, through back-calculation, using deformation measurements
- Nevertheless, there is always uncertainty
- And, natural lithological heterogeneity