Lecture 4: Kinematic Analysis I (Slopes)
Author's Note:

The lecture slides provided here are taken from the course “Geotechnical Engineering Practice”, which is part of the 4th year Geological Engineering program at the University of British Columbia (Vancouver, Canada). The course covers rock engineering and geotechnical design methodologies, building on those already taken by the students covering Introductory Rock Mechanics and Advanced Rock Mechanics.

Although the slides have been modified in part to add context, they of course are missing the detailed narrative that accompanies any lecture. It is also recognized that these lectures summarize, reproduce and build on the work of others for which gratitude is extended. Where possible, efforts have been made to acknowledge the various sources, with a list of references being provided at the end of each lecture.

Errors, omissions, comments, etc., can be forwarded to the author at: erik@eos.ubc.ca
Rock Slope – Continuum or Discontinuum?

In a moderately jointed rock mass, slope failure is generally dictated directly by the presence of discontinuities, which act as planes of weakness within the rock mass. These interact to control the size and the direction of movement of the slope failure.
Discontinuity Mapping

Scanline mapping

Window mapping

Hudson & Harrison (1997)
# Discontinuity Mapping

## DISCONTINUITY SURVEY DATA SHEET

### GENERAL INFORMATION

- **Location**
- **Station / Hole No.**
- **Date**
- **Day**
- **Month**
- **Year**
- **Inspector**
- **Discontinuity data sheet no.**

### NATURE AND ORIENTATION OF DISCONTINUITY

<table>
<thead>
<tr>
<th>Drift / Length</th>
<th>Type</th>
<th>Up Dip</th>
<th>Persistence</th>
<th>Aperture / Width</th>
<th>Nature of Filling</th>
<th>Surface Roughness</th>
<th>Surface Shape</th>
<th>Water Flow</th>
<th>Water Flow (open)</th>
<th>Water Flow (filled)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>0. Fault zone</td>
<td>1.  Very low persistence</td>
<td>1. Very light (&lt;0.1 mm)</td>
<td>1. Clean</td>
<td>51. Very soft clay</td>
<td>0.025</td>
<td>6. Never</td>
<td>0. The water flow along it does not appear possible</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Joint</td>
<td>2. Low</td>
<td>2. Partly open (5-25 mm)</td>
<td>2. Slightly staining</td>
<td>52. Soft clay</td>
<td>0.05-0.10</td>
<td>7. Never</td>
<td>1. The discontinuity is dry with no evidence of water flow</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Concholite</td>
<td>3. Medium</td>
<td>3. Mostly open (25-50 mm)</td>
<td>3. Non-cohesive</td>
<td>53. Firm clay</td>
<td>0.10-0.25</td>
<td>8. Occasionally</td>
<td>2. The discontinuity is dry but shows evidence of water flow, i.e. rust stains, etc.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Schistosity</td>
<td>4. High</td>
<td>4. Wide (10-19 mm)</td>
<td>4. Invasive clay or clay matrix</td>
<td>54. Very soft clay</td>
<td>0.25-0.50</td>
<td>9. Occasionally</td>
<td>3. The discontinuity is damp but no free water is present</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Shear</td>
<td>5. Very high persistence</td>
<td>5. Extremely wide (10-40 cm)</td>
<td>5. Cemented</td>
<td>55. Hard clay</td>
<td>&lt;0.50</td>
<td>10. Occasionally</td>
<td>4. The discontinuity shows seepage, occasional drops of water, but no continuous flow</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Tension Check</td>
<td>6.</td>
<td>6. Extremely wide (&gt;1 m)</td>
<td>6. Chlorite, talc or gypsum</td>
<td>56. Very soft rock</td>
<td>0.5-1.0</td>
<td>11. Continuously</td>
<td>5. The discontinuity shows a continuous flow of water (estimate 1 to 2 meters per minute)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Foliation</td>
<td>7.</td>
<td>7. Very wide (1-10 cm)</td>
<td>7. Other - specify</td>
<td>57. Very weak rock</td>
<td>1.5-4.9</td>
<td>12. Continuously</td>
<td>6. The filling materials are heavily consolidated and dry; significant flow appears unlikely due to very low permeability</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. Bedding</td>
<td>8.</td>
<td>8. Extremely wide (25-100 cm)</td>
<td>8. Other - specify</td>
<td>58. Weak rock</td>
<td>5-10</td>
<td>13. Continuously</td>
<td>7. The filling materials are damp, but no free water is present</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. Ductile</td>
<td>10.</td>
<td>10. Extremely wide (&gt;1 m)</td>
<td>10. Other - specify</td>
<td>60. Very strong rock</td>
<td>100-500</td>
<td>15. Continuously</td>
<td>9. The filling materials show signs of extrusion, continuous flow of water (estimate 10 meters per minute)</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

### Termination

- **0. Neither end visible**
- **1. Drilled end visible**
- **2. Both ends visible**

### Surface shape

- **1. Rough**
- **2. Slickened**
- **3. Flattened**
- **4. Sticking together**

### Surface roughness

- **1. Extremely close spacing (<2 mm)**
- **2. Very close spacing (2-5 mm)**
- **3. Close spacing (5-20 mm)**
- **4. Moderate spacing (20-60 mm)**
- **5. Wide spacing (60-200 mm)**
- **6. Very wide spacing (>200 mm)**
- **7. Extremely wide spacing (>400 mm)**

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Wyllie & Mah (2004)
Remote sensing techniques like LiDAR and photogrammetry, provide a means to collect rock mass data from slopes that would otherwise be inaccessible or dangerous. Discontinuity orientation, persistence, and spacing data can be extracted from the 3-D point cloud of the scanned surface.
Stereonets - Pole Plots

Plotting dip and dip direction, pole plots provide an immediate visual depiction of pole concentrations. All natural discontinuities have a certain variability in their orientation that results in scatter of the pole plots. However, by contouring the pole plot, the most highly concentrated areas of poles, representing the dominant discontinuity sets, can be identified.

It must be remembered though, that it may be difficult to distinguish which set a particular discontinuity belongs to or that in some cases a single discontinuity may be the controlling factor as opposed to a set of discontinuities.
Pole Plots & Modes of Slope Instability

Typical pole plots for different modes of rock slope failure.

Wyllie & Mah (2004)
Discontinuity Persistence

Persistence refers to the areal extent or size of a discontinuity plane within a plane. Clearly, the persistence will have a major influence on the shear strength developed in the plane of the discontinuity, where the intact rock segments are referred to as 'rock bridges'.

<table>
<thead>
<tr>
<th>Description</th>
<th>Modal trace length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>very low persistence</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>low persistence</td>
<td>1–3</td>
</tr>
<tr>
<td>medium persistence</td>
<td>3–10</td>
</tr>
<tr>
<td>high persistence</td>
<td>10–20</td>
</tr>
<tr>
<td>very high persistence</td>
<td>20</td>
</tr>
</tbody>
</table>

Increasing persistence
Discontinuity Persistence

Together with spacing, discontinuity persistence helps to define the size of blocks that can slide from a rock face. Several procedures have been developed to calculate persistence by measuring their exposed trace lengths on a specified area of the face.

Step 1: define a mapping area on the rock face with dimensions $L_1$ and $L_2$.

Step 2: count the total number of discontinuities ($N''$) of a specific set with dip $\psi$ in this area, and the numbers of these either contained within ($N_c$) or transecting ($N_t$) the mapping area defined.

For example, in this case:

$N'' = 14$

$N_c = 5$

$N_t = 4$

Pahl (1981)
**Discontinuity Persistence**

Step 1: define a mapping area on the rock face with dimensions $L_1$ and $L_2$.

Step 2: count the total number of discontinuities ($N''$) of a specific set with dip $\psi$ in this area, and the numbers of these either contained within ($N_c$) or transecting ($N_t$) the mapping area defined.

Step 3: calculate the approximate length, $\bar{l}$, of the discontinuities using the equations below.

\[
m = \frac{(N_t - N_c)}{(N'' + 1)}\]

\[
H' = \frac{L_1 \cdot L_2}{(L_1 \cdot \cos \psi + L_2 \cdot \sin \psi)}\]

\[
\bar{l} = H' \frac{(1 + m)}{(1 - m)}\]

Again, for this case:

If $L_1 = 15 \text{ m}$, $L_2 = 5 \text{ m}$ and $\psi = 35^\circ$, then $H' = 4.95 \text{ m}$ and $m = -0.07$. From this, the average length/persistence of the discontinuity set $\bar{l} = 4.3 \text{ m}$.
Discontinuity Spacing

Spacing is a key parameter in that it controls the block size distribution related to a potentially unstable mass (i.e. failure of a massive block or unravelling-type failure).
Discontinuity Roughness

From the practical point of view of quantifying joint roughness, only one technique has received some degree of universality – the Joint Roughness Coefficient (JRC). This method involves comparing discontinuity surface profiles to standard roughness curves assigned numerical values.
Dilatancy and Shear Strength

In the case of sliding of an unconstrained block of rock from a slope, dilatancy will accompany shearing of all but the smoothest discontinuity surfaces. If a rock block is free to dilate, then the second-order asperities will have a diminished effect on shear strength.

By increasing the normal force across a shear surface by adding tensioned rock bolts, dilation can be limited and interlocking along the sliding surface maintained, allowing the second-order asperities to contribute to the shear strength.
Mechanical Properties of Discontinuities

![Graph showing mechanical properties of discontinuities.](image)

1. Bentonite shale
2. Bentonite seams in chalk
3. Bentonite; thin layers
4. Bentonite; triaxial tests
5. Clay, over consolidated
6. Limestone, 10–20 mm clay infillings
7. Lignite and underlying clay contact
8. Coal measures; clay mylonite seams
9. Limestone; <1 mm clay infillings
10. Montmorillonite clay
11. Montmorillonite; 80 mm clay seam in chalk
12. Schists/quartzites; stratification, thick clay
13. Schists/quartzites; stratification, thick clay
14. Basalt; clayey, basaltic breccia
15. Clay shale; triaxial tests
16. Dolomite, altered shale bed
17. Diorite/granodiorite; clay gouge
18. Granite; clay-filled faults
19. Granite; sandy-loam fault fillings
20. Granite; shear zone, rock and gouge
21. Lignite/marl contact
22. Limestone/marl/lignites; lignite layers
23. Limestone; marlaceous joints
24. Quartz/kaolin/pyrolusite; remolded triaxial
25. Slates; thinly laminated and altered
26. Limestone; 10–20 mm clay infillings
Discontinuity properties can vary over a wide range, even for those belonging to the same set. The distribution of a property can be described by means of probability distributions.

A normal distribution is applicable where a particular property’s mean value is the most commonly occurring. This is usually the case for dip and dip direction.

A negative exponential distribution is applicable for properties of discontinuities, such as spacing and persistence, which are randomly distributed.

A negative exponential function:

\[ f(x) = \frac{1}{\bar{x}} \left( e^{-x/\bar{x}} \right) \]

Wyllie & Mah (2004)
Discontinuity Data - Probability Distributions

From this, the probability that a given value will be less than dimension \( x \) is given by:

\[
F(x) = (1 - e^{-x/\bar{x}})
\]

For example, for a discontinuity set with a mean spacing of 2 m, the probabilities that the spacing will be less than:

- 1 m: \( F(x) = (1 - e^{-1/2}) = 40\% \)
- 5 m: \( F(x) = (1 - e^{-5/2}) = 92\% \)
Structurally-Controlled Instability Mechanisms

Structurally-controlled instability means that blocks formed by discontinuities may be free to slide from a newly excavated slope face under a set of body forces (usually gravity). To assess the likelihood of such failures, an analysis of the kinematic admissibility of potential wedges or planes that intersect the excavation face(s) can be performed.
To consider the kinematic admissibility of plane instability, five necessary but simple geometrical criteria must be met:

(i) The plane on which sliding occurs must strike near parallel to the slope face (within approx. ±20°).

(ii) Release surfaces (that provide negligible resistance to sliding) must be present to define the lateral slide boundaries.

(iii) The sliding plane must “daylight” in the slope face.

(iv) The dip of the sliding plane must be greater than the angle of friction.

(v) The upper end of the sliding surface either intersects the upper slope, or terminates in a tension crack.

Wyllie & Mah (2004)
Kinematic Analysis - Rock Slope Wedge Failure

Similar to planar failures, several conditions relating to the line of intersection must be met for wedge failure to be kinematically admissible:

(i) The dip of the slope must exceed the dip of the line of intersection of the two wedge forming discontinuity planes.

(ii) The line of intersection must "daylight" on the slope face.

(iii) The dip of the line of intersection must be such that the strength of the two planes are reached.

(iv) The upper end of the line of intersection either intersects the upper slope, or terminates in a tension crack.

Wyllie & Mah (2004)
Daylight Envelope: Zone within which all poles belong to planes that daylight, and are therefore potentially unstable.
Friction Cone: Zone within which all poles belong to planes that dip at angles less than the friction angle, and are therefore stable.
Having determined from the daylight envelope whether block failure is kinematically permissible, a check is then made to see if the dip angle of the failure surface (or line of intersection) is steeper than the friction angle.

Thus, for poles that plot inside the daylight envelope, but outside the friction circle, translational sliding is possible.
Pole Plots - Kinematic Admissibility

Wyllie & Mah (2004)
Wedge Failure - Direction of Sliding

Scenario #1: If the dip directions of the two planes lie outside the included angle between \( \alpha_i \) (trend of the line of intersection) and \( \alpha_f \) (dip direction of face), the wedge will slide on both planes.

Example scenario #2: If the dip directions of one plane (e.g., Plane A) lies within the included angle between \( \alpha_i \) (trend of the line of intersection) and \( \alpha_f \) (dip direction of face), the wedge will slide on only that plane.
A rock slope with a history of block failures is to be stabilized through anchoring. To carry out the design, a back analysis of earlier block failures is first performed to obtain joint shear strength properties.
Case History: Rock Slope Stabilization
Case History: Rock Slope Stabilization

Assume: Water in tension crack @ 50% the tension crack height & water along discontinuity.

\[ F = \frac{\text{Resisting Force}}{\text{Driving Force}} = \frac{(W \cos \theta - U) \tan(\phi' + i)}{W \sin \theta + V} = 1.0 \]

\[ V = \frac{1}{2} \gamma_w z_w^2 = \frac{1}{2} (62.4 \text{pcf})(4)^2 = 0.5 \text{ kips} \]

\[ U = \frac{1}{2} L \gamma_w z_w = \frac{1}{2} (10')(4')(62.4 \text{pcf}) = 1.25 \text{ kips} \]

\[ F = \frac{(12.8^K \cos 35^\circ - 1.25^K) \tan(\phi' + i)}{12.8^K \sin 35^\circ + 0.5^K} = \frac{9.24^K \tan(\phi' + i)}{7.84^K} = 1.0 \]

\[ \tan(\phi' + i) = \frac{7.84^K}{9.24^K} = 0.748 \Rightarrow (\phi' + i) = 40.3^\circ \text{ or } 40^\circ \]
Case History: Rock Slope Stabilization

Markland Analysis

Courtesy: B. Fisher (Kleinfelder)
Case History: Rock Slope Stabilization

Given:

- Unstable Rock Slope
- 40 ft tall
- About 55 degrees
- Joint Set Dips 38 degrees
- $\phi^s + i \sim 38 - 40$ degrees

From previous back analysis of failed block below bridge abutment.
Case History: Rock Slope Stabilization

1. “Worst case” tension crack distance is 8.6 ft for a “dry” condition.

2. Assume 50% saturation for tension crack.

3. Estimate “super bolt” tension given desired bolt inclination.

4. Distribute “super bolt” tension over slope face based on available bolts.

5. Make sure and “bolt” all unstable blocks.
Case History: Rock Slope Stabilization

Results:

1. 22 kips tension/ft required at 5 deg downward angle for $F = 1.5$

2. Slope face length is equal to:

$$L_{\text{Face}} = \frac{(H)}{\cos \psi_f} = \left( \frac{40}{\cos 35}\right) = 48.83\text{ft}$$

$$22^K / 48.83\text{ft face} = 0.5\text{ksf/ft face}$$

$$25^K / 0.5\text{ksf} = 50\text{ ft}^2$$

$$\sqrt{50 \text{ft}} = 7.1\text{ft} \approx 7.0\text{ ft}$$

$$7.0\text{ft}(\cos 35) = 5.5\text{ft O.C. using elevation}$$
Case History: Rock Slope Stabilization

Recommendations:

1. 8 rows of bolts (40/5 = 8)
2. Try to bolt every block
3. Grout length determined by contractor
4. Rule of thumb, grout length; UCS/30 < 200 psi adhesion
5. Contractor responsible for testing or rock bolts
6. Engineer responsible to "sign off" on Contractors tests

55 deg slope

40'

Roadway

35-40°
Case History: Rock Slope Stabilization

Courtesy: B. Fisher (Kleinfelder)
Computer-Aided Planar Analysis

Filename: RocPlane2
Project Title: RocPlane - Planar Wedge Stability Analysis

Dist. to Slope Crest: 50.346 m
Upper Face Width: 47.239 m

Upper Face Height: 8.328 m
Upper Face Angle: 10.0°
Driving Force: 4268.50 t/m

Slope Height: 60.030 m
Slope Angle: 50°
Failure Plane Angle: 35°
Normal Force: 2785.68 t/m

Factor of Safety: 0.98

<table>
<thead>
<tr>
<th>External Forces</th>
<th>#</th>
<th>Angle</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Driving Force</td>
<td>1</td>
<td>30.0°</td>
<td>300.000 t</td>
</tr>
<tr>
<td>Resisting Force</td>
<td>2</td>
<td>60.0°</td>
<td>50.000 t</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-20.0°</td>
<td>80.000 t</td>
</tr>
</tbody>
</table>

(Rocscience - RocPlane)
Lecture References


