THESIS SUMMERY

THREE-DIMENSIONAL NUMERICAL MODELLING OF ROCK FAILURE PROCESS

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1 Introduction

Analysis of a wide range of problems in rock mechanics and engineering requires knowledge of the failure process in rock. This includes tunnel design, rock slope design and other engineering applications as well as geophysical problems such as earthquake prediction. Brittle fracture process is a failure phenomenon that has been intensively investigated in rock physics and geotechnical engineering.

Though experimental observations have provided a great deal of insight into the complicated failure process (Wawersik and Fairhurst, 1970; Wawersik and Brace, 1971; Lockner, et al., 1992; Hudson, 1993; Pells, 1991 and 1993; Van Mier, 1997; Hart and Cundall, 1998; Brady, 1993; Fairhurst, 1998), the mechanism of rock fracture under mechanical loading, the details of the failure mechanisms, including the microfracture initiation, propagation, and coalescence, is not fully understood. One of the key questions is: when can the incipient fracture plane be recognized during the loading stage of an initially intact rock, or at what point does fracture interaction overwhelm the local variations in the stress field or in the local properties and drive the system to fracture, either by coalescence of fractures or by extension of one fracture (Lockner, et al., 1992; Blair and Cook, 1997)? Moreover, some of the experimental studies concerned the rock fracture and the complete failure process at the surface, it mainly because that rocks are non-transparent and it is difficult to trace the propagation of the rock fracture and fragmentation within the rock. The evolution of the fracture progressive process cannot be successively and visually shown in experimental observations. Besides, it is too expensive to conduct a large number of experiments.

The use of analytical methods developed based on linear-elastic fracture mechanics for a single or multiple cracks (holes) with ideal shapes (Hori and Nemat-Nasser, 1985 and 1986) is difficult or even arduous (Schlangen and Garboczi, 1992 and 1997). Moreover, the analytical models have to be simplified and sometimes this simplification overlooks important factors influencing the material behaviour, such as heterogeneities and discontinuities on different scales. From this point of view, numerical analysis seems be the best that can be achieved. The appearance of personal computer and gradually mature computing methods make it possible to simulate the complete rock failure process by numerical methods.

Many researchers have applied FLAC to simulate the brittle fracture in rocks (McKinnon and Barra, 1998; Fang and Harrison, 2002). Recent advances in the BEM and the displacement discontinuity method (DDM), based on BEM, have resulted in a new generation of the method for the solution of crack problems in rock fracture (Tan et al., 1997 and 1998I Blair and Cook, 1998; Shen et al.,2004). The DEM method was originally created as a two-dimensional representation of a jointed rock mass by Cundall (1971), but has been extended to applications in studies on microscopic mechanisms in granular material (Cundall and Strack, 1983), and crack development in rock and concrete (Plesha and Aifantis, 1983; Lorig and Cundall, 1987).

FEM may be the most mature numerical method applied to model rock failure process. At present a lot of well-known commercial codes are available, such as ANSYS, ABAQUS, MSC.NASTRAN, FEMLAB, ADINA and ALGOR. Saouma and Kleinovsky (1984) simulated crack initiation and subsequent crack propagation in the process of major chip formation with their finite element analysis (FEA) program. Swenson and Ingraffea (1988) use a finite element code to model the dynamic propagation of a discrete crack. Wawrzynek and Ingraffea (1989) studied fracture problems using a two-dimensional crack propagation simulator developed on the basis of the FEM. Alehossein and Hood (1996) used a finite element code to simulate crack problems in indentation.

Some other rock fracture models, such as the energy model (Karihaloo, 1984; Shen and
Stephansson, 1994), the stress model (Paul and Mirandy, 1976; Tirosh and Catz, 1981), the lattice model (Schlangen and Van Mier, 1992; Schlangen and Garboczi, 1997; Place and Mora, 1999), the cellular automaton model (Wilson et al., 1996; Psakhie et al., 1999) and so on, have been developed to accompany the numerical methods.

Among all the numerical methods mentioned above, statistical modeling has emerged to be a promising technique for analysis of fracture in heterogeneous materials such as rock (Blair and Cook, 1998). The combinations of statistical theory with numerical models such as the lattice model (Van Mier, 1997) the bonded particle model (Cundal, 1998; Hart and Cundal, 1998) or the RFPA2D model based on FEM (Tang, 1997) are found to be appropriate for modeling brittle materials such as rocks. 

At present, lattice models do not seem well suited for modeling compressive fracture. However, by means of the UDEC model or other models such as the rigid particle model, better results are obtained, although with more complex meso-level material laws (Cundal, 1998). Moreover, neither particle model nor lattice model has considered the interaction and coalescence between the crack systems induced by external loading.

However, due to high computational costs and the complicated fracture mechanism, most of these models have been generally developed in 2D. Few presentations can be found to explain the progressive failure of rock in 3D, including 3D crack initiation, propagation, and coalescence following 3D spatial fracture path resulting from material heterogeneities.

The two-dimensional analysis is intrinsically limited. There are many loading cases which cannot be simulated in 2D (for instance biaxial loading cases which fail out of plane, or triaxial tests), and even for those which can, it would be desirable to evaluate the importance of the three-dimensional effect because, strictly speaking, 2D calculations would correspond to arrangements of aggregates or particles of prismatic shape in the third dimension (Caballero et al., 2005). Further, tension experiments on concrete have shown that the fracture process is rarely uniform in the third direction, since cracks generally propagate from one corner, rather than uniformly through the entire depth of a specimen (Van Mier, 1999). In addition, neither in plane strain nor in axisymmetric triaxial loading conditions can the intermediate principal stress be taken into consideration. Many results from the tests both under plane strain and axisymmetric triaxial loading conditions show an obvious difference. Many experimental studies shows that higher strength values were observed under plane strain loading and are believed to be due to the strengthening effect of the intermediate principal stress (Mogi, 1967 and 1979; Handin et al., 1967; Nascimento et al., 1974; Michelis, 1987; Yumlu and Ozbay, 1995). The strength increases considerably in excess of its corresponding value for standard triaxial tests with an increasing value of the intermediate principal stress (Michelis, 1987; Yumlu and Ozbay, 1995; Yu, 2002).

Although recently some have started offering 3D results as well, in order to reduce the computational time, the adopted meshes are usually too coarse and as far as that concerned the numerical simulations consisting more than 100,000 elements are not found. Due to the enormous increase of number of degrees of freedom, numerical model implemented in a finite element code for parallel computing is necessary to limit the computing time in rock failure simulations.

RFPA2D model have been successfully used to model the failure of brittle materials and the associated microseismicities. It is further developed to model the strata movement in mining (Liu, 2001), the concrete failure (Zhu, 2001), and the coupling of flow and solid (Liu et al., 2000; Yang, 2001; Tang et al., 1997-2002).

In this thesis, firstly a three-dimensional numerical model is proposed on the basis of the parallel computing method and the three-dimensional mesoscopic elastic damage model. The main contents
of the RFPA\textsuperscript{3D} code are the heterogeneous material model, the modified failure criteria, mesoscopic mechanical model, and the parallel computing. Then the developed RFPA\textsuperscript{3D} code is used to investigate the three-dimensional rock failure progressive process in various mechanical conditions by conducting a serial of numerical tests. Some mechanical failure mechanisms and some significant factors that affect the failure of rocks, such as heterogeneity, specimen shape (slenderness), size and boundary restraint, et al., are discussed by using three-dimensional numerical studies. The numerical results show that the RFPA\textsuperscript{3D} model can reproduce the fracture processes observed in real physical experiments, and some fracture mechanisms that can not be interpreted in two-dimensional modeling, such as fractal characteristic during fracture process, the three-dimensional fracture spacing and associated pattern transition and the intermediate principal stress effect on rock failure under true triaxial compression, are also discussed in the thesis.

2 Description of the numerical model

2.1 Mesh generation and implement of heterogeneity

![Figure 2.1](image)

Figure 2.1  Mesh generation in RFPA\textsuperscript{3D}. Heterogeneity is introduced into the numerical specimen by following a statistical distribution. However, the mechanical properties in each element are homogeneous.

As shown in Figure 2.1, the model in RFPA\textsuperscript{3D} is meshed into brick elements with randomly distributed mechanical properties. In order to deal with real random microstructures in numerical simulation, rock heterogeneity can be characterized better with statistical approaches (Liu, 2001). In RFPA\textsuperscript{3D}, since the numerical specimens consist of the elements with the same shape and size, there is no priority geometrically in any orientation in the specimen. Disorder can be obtained by means of randomly distributions of the mechanical properties of the elements. The statistical distribution of elastic modulus can be described by Weibull distribution function (2.1), even distribution function (2.2) or normal distribution function (2.3).

\[
W(x) = \frac{m}{x_0}(\frac{x}{x_0})^{m-1}\exp\left[-\left(\frac{x}{x_0}\right)^m\right] \\
E(x) = \begin{cases} 
0, & x \leq a \\
1/(b-a), & a \leq x \leq b \\
1, & x \geq b 
\end{cases} \\
N(x) = \frac{1}{\sqrt{2\pi}s}\exp\left[-\frac{(x-x_0)^2}{2s^2}\right]
\]

(2.1) \hspace{3cm} (2.2) \hspace{3cm} (2.3)

2.2 Constitutive law

It was assumed that each element keeps linear elastic before failure, and it may fail in either in tensile failure mode or shear failure mode. If the elemental stress state satisfies the failure criterion, the tensile failure criterion or shear failure criterion, the element fails and the elastic modulus reduces to a small value and its strength falls to residual tensile strength.

The damage variable $D$ can be described as following when the element is subjected to uniaxial..
tension:

\[
D = \begin{cases} 
0 & (\varepsilon_3 > \varepsilon_{0}) \\
1 - \frac{\sigma_{n}}{E_0} & (\varepsilon_{10} \geq \varepsilon_3 \geq \varepsilon_u) \\
1 & (\varepsilon_3 \leq \varepsilon_u)
\end{cases}
\quad (2.4)
\]

where $\sigma_n$ and is the residual strength of the element, and $\sigma_n = -\lambda |\sigma_1|$. $\varepsilon_{10}$ is the tensile strain at the point of failure. $\varepsilon_u$ is the ultimate tensile strain and can be described as $\varepsilon_u = \eta \varepsilon_{10}$. $\eta$ is the ultimate tensile strain coefficient and $\lambda$ is coefficient of residual strength.

Compressive softening induced by shear damage at mesoscopic level is also assumed to exist when the mesoscopic element is under compressive and shear stress. In shear failure mode, the damage variable $D$ can be described as follows:

\[
D = \begin{cases} 
0 & \varepsilon_1 < \varepsilon_{10} \\
1 - \frac{\sigma_{n}}{E_0 \varepsilon_1} & \varepsilon_1 \geq \varepsilon_{10}
\end{cases}
\quad (2.5)
\]

where $\sigma_{n}$ is the peak strength of the element subjected to uniaxial compression and $\sigma_{10}$ is the compressive stress at the point of shear failure. If the specimens are subjected to three-dimensional stress loading, according to Mazars (1984) investigation, we can extend it from one dimensional damage model to a three-dimensional model by using an equivalent strain $\bar{\varepsilon}$ instead of the uniaxial tensile strain or compressive strain in (2.4) and (2.5).

\[
\sigma_{ij} = \begin{cases} 
\frac{\sigma_{n}}{E_0} (2G \varepsilon_{ij} + \lambda \delta_{ij} \varepsilon_{kk}) & (\bar{\varepsilon} > \varepsilon_{10}) \\
\frac{\sigma_{n}}{E_0} \left( \varepsilon_{ij} + \frac{\delta_{ij} \varepsilon_{kk}}{1+\nu(1-2\nu)} \right) & (\varepsilon_{10} \geq \bar{\varepsilon} \geq \varepsilon_u) \\
0 & (\bar{\varepsilon} < \varepsilon_u)
\end{cases}
\quad (2.6)
\]

\[
\sigma_{ij} = \left[ \begin{array}{c}
\frac{\sigma_{n}}{E_0} (2G \varepsilon_{ij} + \lambda \delta_{ij} \varepsilon_{kk}) \\
\frac{\sigma_{n}}{E_0} \left( \varepsilon_{ij} + \frac{\delta_{ij} \varepsilon_{kk}}{1+\nu(1-2\nu)} \right)
\end{array} \right] 
\quad (2.7)
\]

Figure 2.2 The stress-strain relation of the element for two different failure modes.

According to the elastic damage constitutive theory, the stress-strain relation of the element subjected to complex three-dimensional stress in these two different failure modes can be obtained as following functions:

\[
\sigma_{ij} = \left[ \begin{array}{c}
(2G \varepsilon_{ij} + \lambda \delta_{ij} \varepsilon_{kk}) \\
(2G \varepsilon_{ij} + \lambda \delta_{ij} \varepsilon_{kk})
\end{array} \right] 
\quad (2.6)
\]

\[
\sigma_{ij} = \left[ \begin{array}{c}
(2G \varepsilon_{ij} + \lambda \delta_{ij} \varepsilon_{kk}) \\
(2G \varepsilon_{ij} + \lambda \delta_{ij} \varepsilon_{kk})
\end{array} \right] 
\quad (2.7)
\]
where \( G = \frac{E_0}{2(1 + \nu)} \) \( \lambda = \frac{E_0\nu}{(1 + \nu)(1 - 2\nu)} \) \( \delta_{ij} = \begin{cases} 1, & (i = j), \\ 0, & (i \neq j). \end{cases} \)

However, we can just obtain the mechanical parameters of the macroscopic specimen for a specific rock specimen in laboratory experiments, and it is impossible to obtain the parameters of the elements. For Weibull distribution, a parametric study can be performed to obtain the relationships between the macroscopic parameters (compressive strength \( \sigma_c \), elastic modulus \( E_0 \)) of the specimen and the seed parameters (mean value of compressive strength \( \bar{\sigma}_c \), and elastic modulus \( \bar{E}_0 \)) of mesoscopic elements by using the linear least squares technique.

\[
\bar{\sigma}_c = [a_1 \ln(m) + b_1] \sigma_c \tag{2.8}
\]

\[
\bar{E}_0 = [a_2 \ln(m) + b_2] E_0 \tag{2.9}
\]

The uniaxial compressive strength and elastic modulus corresponding to the specific rock can be obtained from the laboratory test. The shape parameter \( m \) can be obtained as a matter of experience from experimental and numerical tests (Wong, et al., 2006).

**2.3 Failure criterion**

In three-dimensional studies, the traditional Mohr-Coulomb strength criterion or Hoek-Brown strength criterion is not valid any longer since the effect of intermediate principal stress is not taken into consideration. In addition to Mohr-Coulomb strength criterion and Hoek-Brown strength criterion, the Drucker—Prager strength criterion and Unified Strength Criterion proposed by Yu (2002), which consider all of the three principal stresses, are also provided in RFPA3D. The criterions mentioned are all of shear failure criterions, and the tensile failure is not considered. As we know, many fractures are induced by tensile stress and even some shear failures are found in the shear zones. Therefore, the maximal tension strength criterion is introduced into the shear failure criterions in RFPA3D.

Twins Shear Criterion with tension cut-off can be expressed as follows:

\[
\begin{align*}
F &= a\sigma_1 - \frac{1}{2} (\sigma_2 + \sigma_3) - \sigma_t \geq 0 & \sigma_1 \leq \sigma_c + a\sigma_t \frac{1 + a}{1 + a} \\
F^* &= a\sigma_1 - \frac{1}{2} (\sigma_2 + \sigma_3) - \sigma_t \geq 0 & \sigma_2 > \sigma_c + a\sigma_t \frac{1 + a}{1 + a} \\
\end{align*}
\]

In order to consider the influence of the intermediate principal stress more explicitly, a parameter \( b \) is introduced into the Unified Strength Criterion, which can be expressed as follows:

\[
\begin{align*}
F &= a\sigma_1 - \frac{1}{1 + b} (b\sigma_2 + \sigma_3) - \sigma_t \geq 0 & \sigma_1 \leq \sigma_c + a\sigma_t \frac{1 + a}{1 + a} \\
F^* &= a\sigma_1 - \frac{1}{1 + b} (b\sigma_2 + \sigma_3) - \sigma_t \geq 0 & \sigma_2 > \sigma_c + a\sigma_t \frac{1 + a}{1 + a} \\
\end{align*}
\]

When \( b = 1 \), the Unified Strength Criterion can be simplified into Twins Shear Criterion.

**2.3 AE and AE energy**

It has been proved that there is a unique association between a single AE event and a micro-crack forming event. It is assumed that the number of seismic events or AE is proportional to the number of
failed elements. Thus, by recording the counts of failed elements, the AE associated with the progressive failure process can be simulated. Moreover, in a brittle or quasi-brittle material such as rock, AE is predominantly related to the release of elastic energy (Tang, 1997). The energy released of the failure element can be determined by the difference of the strain energy stored in the element before failure and after failure. By using a three-dimensional graphic library, OpenGL, each AE event can be de presented as a 3D ball in the post processing picture. The radius of the ball presents the relative energy released. Since the element may fail in two different modes, two different colors are selected to draw the balls to distinguish the failure modes of each element as well as the fracture mode of the numerical model subjected to stress loading.

2.4 Loading procedure

In order to perform the finite element analysis, the parameters of the numerical model, such as shape parameter, mean value of the strength, and mean value of the elastic modulus et al., are specified, and the initial boundary conditions are applied to it. After that, an external stress or displacement disturbance is applied to the numerical model caused by force loading, displacement loading or stress redistribution. A finite element stress analyzer is used to calculate the stress and strain distributions in the finite element network. The calculated stresses are substituted into the strength criterion to check whether or not elemental damage occurs. If the strength criterion is not satisfied, the external force loading or displacement loading is increased further. Otherwise, the element is damaged and becomes weak according to the rules specified by the mesoscopic elemental mechanical model for elastic damage, which results in a new perturbation. The stress and deformation distributions throughout the model are then adjusted instantaneously after each rupture to reach the equilibrium state. Due to the new stress disturbances, the stress of some elements may satisfy the critical value and further ruptures are caused. The process is repeated until no failure elements are present. The external load is then increased further. In this way the system develops a macroscopic fracture. Due to the interaction induced by stress redistribution and long-range deformation, a single important element failure may cause an avalanche of additional failures in neighboring elements, leading to a chain reaction releasing more energy.

2.5 Implement of the RFPA3D code

There are four modules to implement the RFPA3D code: preprocessing module, FEM module, failure analysis module and postprocessing module.

In order to make its use more convenient, a user-friendly pre- and post-processor is also developed and integrated into the RFPA3D code using Visual C++ (VC) to generate the finite element mesh, prepare the input data, and visually display the analyzed results. VC is a powerful Object-Oriented Programming (OOP) Integrated Development Environment (IDE) tool for building 32-bit applications in a Windows environment. With the aid of Windows Graphic Device Interface (GDI), it is easy to develop a module for the RFPA3D code to design the geometrical models for mechanical tool and rock with various irregular shapes, and then fill them with different material properties. GDI also helps make the model results displayed graphically as an animation to assistant users in understanding the rock failure mechanisms.

The FEM modules for Personal Computer (PC) and cluster computers (parallel computing) are totally different. FEM module for PC is developed with FORTRAN 95 language only, while FEM module for cluster computers is developed with FORTRAN 95 language and MPI (Message Passing Interface). MPI is a free library for parallel computing program development. FEM is implemented with FORTRAN 95 language, which can provide a high efficiency to perform FEM calculation. Especially, the strategy of dynamic array helps save memory, which is much in urgent need in
three-dimensional FEM calculating for large scale models.

2.6 Parallel computing

A three-dimensional parallel FEM module and the interface between Linux and Windows XP modules are developed and successfully applied to the analysis of rock progressive failure process.

The main contents of the parallel FEM module in RFPA3D is the parallel code developed with MPI (Message-Passing Interface) on the Linux Redhat9.0 platform. With the aid of MPI/CH, the parallel FEM module in RFPA3D is developed in the Linux Redhat9.0 operation system platform by using FORTRAN90 language and C++ language.

A slaver-master technique is adopted for data distribution and communication in the FEM module in RFPA3D code (Figure 2.3). Firstly the data prepared by the pre-processing module are sent to the console node which serves as a master node, and then the total domain is partitioned into many sub-domains as even as possible and the sub-domains are distributed to many computer nodes which serve as the slaves by using an algorithm termed Domain Decomposition Method (DDM). Each slave computes the assigned sub-domain by an independent FEM program. The processors calculating the sub-domains can exchange information between each other and the total disequilibrium force is calculated. The iterative calculating will continue until the total disequilibrium force decreases to a value as small as enough. The calculated results will be returned back to the master node if all slave nodes have finished their tasks. In the FEM module, PCG (Preconditioning Conjugate Gradient Method) is used to improve iteration performance greatly in resolving the linear functions.

![Diagram](image)

Figure 2.3 A slaver-master technique is adopted for data distribution and communication in the FEM module in RFPA3D code. (a) Sketch map of master-slave strategy. (b) A specimen containing a hole is decomposed into 32 processes. (c) The calculated stress distribution of the numerical specimen discretized into 1,000,000 elements

In order to take advantage of the high computing performance and provide a user-friendly interface, the preprocessing module, failure analysis module and postprocessing module are developed by Microsoft Visual C++ on Microsoft Window XP operation system, while the FEM module is developed by MPI/CH on Redhat9.0 operation system. The Server-Client mode is adopted to develop the interface between the FEM module on Redhat Linux 9.0 and the modules on Windows XP. The numerical results show that for the analysis of the three-dimensional irregular and complicated rock fractures, the parallel computing code is effective and reliable. Moreover, regardless that the scale of computing degree of freedom is from 100,000 to nearly 10,000,000, and the number of processes (CPUs) is from 8 to 64, the parallel program don't need to be modified. This shows that the parallel computing performance of the FEM module is stable, robust and extensible.
3 Numerical Tests on Rock Failure Process to Validate the Model

In this part, serials of numerical simulation are carried out using RFPA3D to calibrate the model. The simulated crack initiation and propagation as well as the whole progressive fracture process are compared with the theoretical results, the experimental observations and other numerical simulation results.

3.1 Homogeneous model subjected to uniaxial tests

Two numerical specimens with homogeneous mechanical properties are prepared to conduct uniaxial compressive loading test and uniaxial tensile loading test respectively. The mean value of the compressive strength is assumed to be 100MPa, and the mean value of the tensile strength is assumed to be 10MPa. Since the numerical specimens are homogeneous, theoretically the peak strength of the specimens will be equal to the mean value of all the elements. According to the complete stress-strain curves (Figure 3.1), the numerical results have a good agreement with the assumed value.

![Complete stress-strain curves obtained by conducting uniaxial tests on homogeneous specimens. The macro response of the homogeneous specimens is much similar to the elemental response at mesoscopic scale.](image)

3.2 Tension tests for heterogeneous models

A numerical specimen with heterogeneous mechanical properties is prepared for tensile strength test. The mean values of uniaxial tensile strength, elastic modulus, and Poisson ratio are assumed to be 10MPa, 20000MPa, and 0.18 respectively. The shape parameter is assumed to be 2.5. All the parameters are determined according to Chen’s study. The size of the specimen is 40mm×40mm×40mm, and it is meshed into a 40×40×40 elemental network.

According to Weibull’s theory, if the distribution of the elemental strength of the heterogeneous specimens follows the Weibull distribution function, the complete stress-strain curve of the idealized model subjected to uniaxial tensile stress can be obtained as follows:

\[
\sigma = E\varepsilon \exp \left(-\frac{\varepsilon}{\varepsilon_0}\right)^a
\]  

Figure 3.2 shows the comparison of the complete stress-strain curves obtained from numerical results and theoretical analysis. It can be found that the numerical results agree well with the analytical solution before the peak strength in the curves. However, the stress from numerical simulation has a higher value than theoretical analysis. This maybe results from the elemental residual strength in the numerical model, while in the theoretical model, the failure elements will lose all their bearing capacity.

It should be noted that the curves are normalized, so the peak strength comparison between the numerical model and the theoretical model will make sense. We can derive the following function from above:
\[
\frac{d\sigma}{d\varepsilon} = E \exp[-(\frac{\varepsilon}{\varepsilon_0})^n][1-m(\frac{\varepsilon}{\varepsilon_0})^n]
\]

(3.2)

If we assume \( \frac{d\sigma}{d\varepsilon} = 0 \), we can get the peak strength of the specimen as follows:

\[
\sigma_{\text{max}} = E\varepsilon_{\text{max}} \exp[1 - (\frac{\varepsilon_{\text{max}}}{\varepsilon_0})^m]
\]

(3.3)

We can rewrite the function above as follows:

\[
\sigma_{\text{max}} = E\varepsilon_0 \left(\frac{1}{m}\right)^{\frac{1}{m}} \left(1 - \frac{1}{m}\right)^{\frac{1}{m}}
\]

(3.4)

If we input the mechanical parameters into the function, we can obtain the tensile strength of the specimen in the following function theoretically:

\[
\sigma_{\text{max}} = 3.44 \text{ MPa}
\]

The peak strength of the numerical specimen is 3.54MPa, which is a little higher than the peak tensile strength obtained from the theoretical model.

**Figure 3.2** Comparison between simulated complete stress-strain curve and theoretical results

**Figure 3.3** Comparisons of simulated complete stress-strain curve with other studies

Hudson (1972) and Zhu (1985) have investigate the uniaxial tensile failure of black granite, marble and sand rock and obtained the complete stress-strain curves by experimental study, and Chen (2002) has also investigate the tensile failure by numerical study. Figure 3.3 shows the comparisons of RFPA\textsuperscript{3D} results with their studies. RFPA\textsuperscript{3D} results agree better with the results of sand rock and marble in the soften stage. The difference between the simulated results and the black granite results from the residual strength in the numerical model and the unsuitable shape parameter for the granite, which is more brittle than both sand rock and marble.

(a) Symmetrical three-point bending test conducted by Landis and asymmetrical three-point bending test conducted by Xeidakis.
Numerical results obtained by using RFPA3D

Figure 3.4 Fracture patterns from experimental observations and numerical tests in symmetrical three-point bending test and asymmetrical three-point bending test.

3.3 Three-point bending tests

Figure 3.4 shows the fracture patterns from numerical tests and experimental observations (Landis, 1999; Xeidakis, 1997) in symmetrical three-point bending test and asymmetrical three-point bending test, respectively. In the symmetrical three-point bending test, tensile cracks are firstly initiated from the tip of the pre-fabricated notch and then propagate forward to the central loading supporting points following the tortuous path. In the asymmetrical three-point bending test, at the beginning, the crack is initiated from the tip of the pre-fabricated notch and propagates towards the midpoint of the specimen, but afterwards it turns upwards becoming sub-parallel to the vertical direction. The simulated fracture patterns have a good agreement with experimental results.

![Fracture patterns](image)

Figure 3.5 The comparison of the peak loading in asymmetrical three-point bending test by varying $L_1$ between the experimental results and the numerical results.

Xeidakis (1997) and Zhu (2001) investigated different fracture patterns as well as the peak loading by varying the distance of loading point and the pre-fabricated notch $L_1$. The comparison of the peak loading by varying $L_1$ is shown in Figure 3.5. All the results show that all the curves have the same trend and the peak loading increases with the increase of $L_1$.

3.4 Comparison with RFPA2D

RFPA2D has been proved to be a valid and useful tool to study the rock failure process in many studies (Tang, 1997, 1998, 2000a, 2000b, 2002, and 2006, et al.; Liu, 2001 and 2002; Zhu, 2001, et al.; Wong, 2006). Comparisons are also made between RFPA3D and RFPA2D. A plane stress model prepared by RFPA2D is meshed into a 40 × 40 network, and a three-dimensional model prepared by RFPA3D is meshed into 40 × 40 × 40 network. Both of them are subjected to uniaxial compression loading and uniaxial tension loading. The normalized stress-strain curves are shown in Figure 3.6. Even though they have the same tendency in the whole failure process, the curves obtained by using RFPA3D exhibit a smoother feature, while the curves simulated by RFPA2D rise or fall in a stepwise way. This may be caused by the “parallel effect” in three-dimensional models, in which a single element failure will not lead to too much stress fluctuation of macro response (as shown in Figure 3.7).
Moreover, a single plane model is prepared by using RFPA3D, and the simulated curve is compared with the results simulated by RFPA2D. The curve of the 3D model appears to be as fluctuant as the 2D model in the soften stage (as shown in Figure 3.8).

![Figure 3.7 “Parallel effect” in three-dimensional models compared with plane stress models.](image)

**4 Applications of RFPA3D code in Rock Failure Process**

In this part, the RFPA3D code is applied in numerical tests for rock failure process. It should be noted that that even though some numerical experiments are undertaken under uniaxial compression or uniaxial tension, due to the heterogeneous physical-mechanical properties distributed throughout the numerical specimens, the deformation localization and the fracture localization, the stress in the specimens are not uniformly distributed and the specimens may have a complex 3D stress state, which can be shown in the following sections.

**4.1 End constraint effect**

Theoretically, material properties should not depend on the specimen geometry and the test conditions, but for many rock property measurements this is apparently not the case (Hudson et al., 1993). In this section, five specimens of the same dimensional size but with different loading platens in terms of modulus is undertaken uniaxial compression to examine the end constraint effect. The value $E_p$ indicates the Young’s modulus of the loading platens at the top and bottom edges of the specimen. $E_p/E_s$ is the ratio of Young’s modulus of platen to specimen. Due to the elastic mismatch between the platen and the specimen, the lateral deformation in platen and specimen is different. As
a result, there will be friction between platen and specimen. This friction may develop either conining compressive stress at the specimen ends if the ratio of platen modulus to specimen modulus $E_p/E_s > 1$ (stiffer constraint, as shown in Figure 4.2(a)), or lateral tensile stress at the specimen ends if the ratio of platen modulus to specimen modulus $E_p/E_s < 1$ (softer constraint, as shown in Figure 4.2(b)). These effects would be at a maximum at the ends of the specimen and taper off from the ends toward the specimen center. Therefore, two cones of compression zones will develop in the specimen with stiffer constraint and fewer fractures are expected in these cone-shaped zones. On the other hand, two cones of tension zones will develop in the case of softer constraint and more fractures are expected, which results in a clear splitting failure mode in the specimen.

The simulations for different platen conditions show that the type of fracture is markedly sensitive to the platen or end conditions. The crack patterns show that almost-vertical splitting cracks develop in specimens loaded with softer loading platens (as shown in Figure 4.2(a)), and the well-known hour-glass failure mode develops in specimens loaded with stiffer loading platens (as shown in Figure 4.2(c, d)).

In addition, the results presented here indicate that a more ductile response is simulated and the peak strength increases when the end constraint increases. It is interesting to find that, although small, the variation of the peak stress is strictly relative to the end constraint conditions.

![Figure 4.1](image1.png)

**Figure 4.1** Rock specimens have a complicated triaxial stress state in the end zones due to the mismatch between the end plates and the specimens.

![Figure 4.2](image2.png)

**Figure 4.2** Fracture patterns are influenced by the end constraints (RFPA3D results)

In theory, uniaxial compressive testing should involve subjecting the test specimen to a uniform uniaxial stress field. However, this never occurs because of constraints applied to the specimen ends by the loading platens. The numerical result also implies that platens made of the same material as the test specimen are more suitable for the tests. Theoretically, the correct way of loading rock
specimens is by means of loading platens that have the same material constants throughout the loading process. The numerical result presented in this paper also implies that platens made of the same material as the test specimen are more suitable for the tests. In fact, a more practical suggestion is that the higher specimens be selected to reduce the ends-effects caused by the mismatch of the loading and tested materials.

4.2 Geometry effect

The geometry of the specimen can influence the strength characterization in several ways (Van Mier, 1997). Here we introduce a parameter $t$ to describe the ratio of the depth to width / thickness of the five specimens. It should be noted that the values of the width of the specimens are equal to the thickness. The ratios of the depth to length / thickness range from 0.5, 1.0, and 2.0 to 3.0. It means that the specimen with $t=3.0$ is the most slender one among these specimens.

![Plots of failure state and AE distribution for specimens with different geometry in idealized loading condition (RFPA results)](image)

Figure 4.3 Plots of failure state and AE distribution for specimens with different geometry in idealized loading condition (RFPA results). The AE events are denoted by the balls with different diameters, which are relative to the energy release during fracture. The colors of the ball denote the failure modes.

The computed stress distributed in the specimens is influenced by the geometry of the specimens. It is shown that the interaction between the stress fields of the two cone shape areas in the ends of the specimen become stronger as the ratio of the height to width of the specimen becomes smaller. The effect of slenderness is clearly reflected in changes in fracture pattern, as demonstrated in Figure 4.3. The failure modes for different ratios of height to width are quite realistic, compared with the experiment tally observed modes reported by Hudson et al. (1993) and Van Mier (1997). Longer specimens show a more dominant splitting failure mode (as seen in Figure 4.3(d) for specimen with $t=3.0$). On the other hand, short specimens show more complicated failure modes: although surface splitting occurs first, more internally inclined fractures in faulting dominate the failure process of the specimen (as seen in Figure 4.3a for specimen with $t=0.5$). When the specimen slenderness increases,
the specimen surface instability is enhanced, its stress state becomes more homogeneous in the
medium of the specimen and failure patterns change; axial splitting occurs instead of faulting.
Increasing \( t \), the failure becomes more brittle. For short specimens, a large number of macrofractures
occur well before the peak stress is reached and a more ductile failure pattern is simulated. For
longer specimens it is found that for specimens with \( t=3 \) the macrocracking starting in the middle
of the specimen does not depend on the contact conditions. This is due to the diminished influence of
the end effects and, more essentially, to the increasing surface instability. The failure pattern clearly
indicates that the splitting failure dominates.

Figure 4.4 (b) plots the peak strength as a function of height to width. The numerical simulations
show that peak stress depends strongly on specimen height. The peak stress sustained by a specimen
decreases with increasing slenderness of the specimen. The lowest specimen gives the highest
strength. It is well known that experimental results also indicate that measured strength decreases
with increasing height to diameter ratio (Kotsovos, 1983).

As well as peak stress, the ratio of specimen height to width also affects the strain softening
behavior of rock. The pre-peak portion of the stress-strain curves shows no significant dependence
on slenderness; however, the post-peak curves are highly dependent on the ratio of specimen height
to width. The numerical simulations of the five specimens reveal that increasing the ratio of height to
width results in a steeper slope of the descending branch, even leading to a severe stress drop. The
numerical results confirm the observation obtained from experiments carried out by Van Mier (1993),
namely, the softening branch becomes steeper with increasing specimen height.

![Graphs showing stress-strain curves and peak strength vs. specimen geometry parameter](image)

**Figure 4.4** (a) Complete stress-strain curves for numerical specimens with different geometry under idealized loading conditions. (b) Relation of peak strength and the specimen geometry parameter \( t \). When \( t>2.5 \), the peak strength trends to a certain value. The ratio of the height to the width is suggested to be from 2.5 to 3 by ISRM (Kovari et al., 1983).

Theoretically, the specimen length would have no effect if the behavior of a rock specimen reflects
only the material characteristics of rock. However, due to the friction between the loading platens
and the specimen, rock specimens have a complicated triaxial stress state in the end zones. The rock
behavior cannot be independent of the distance to the end of the specimen. Therefore, investigators
tend to eliminate the contact effect so that only the vertical stress influence should remain. It can be
achieved by changing the ratio of height to diameter. It has been established that microfracturing
starts in the middle specimen portion (out of the cones at the ends) if the ratio of height to width is
larger than 2 and will be better larger than 3 (Andreev, 1995).

**4.3 Size effect**

Even though mechanical heterogeneities can be considered in Weibull theory, the stress
redistribution resulting from crack propagation can not be taken into account. These led to the development of a new deterministic energetic theory (Bazant et al., 1984, 1987, 2000, 2002, 2005), in which the size effect is explained by stress redistribution due to the development of cracks or occurrence of other discontinuities. In RFPA3D modeling, both the heterogeneity and the stress redistribution considered in deterministic energetic theory are taken into account.

Five single-edge-notched specimens are prepared, and the mechanical parameters of them are all the same. The specimens are geometrically similar to each other, but have different sizes by varying the sides. The size of the five specimens are 200mm×200mm×40mm, 100mm×100mm×20mm, 80mm×80mm×16mm, 40mm×40mm×8mm, and 20mm×20mm×4m, respectively. The fracture process and the complete nominal stress-strain curves are obtained. It is found that, although the constitutive law for the individual element in the numerical model is nearly brittle, a substantial non-linearity exists before the maximal stress. The shapes of the complete stress-strain curves in tension are similar to each other to some extent. However, smaller specimens have a more ductile failure behavior and the loading capacities decrease more gradually compared to the larger specimens. Furthermore, it takes more loading steps for the small specimens to reach the peak loading point and the residual strengths for smaller specimens are higher after the peak point.

![Stress-strain curves](image1)

**Figure 4.5** Stress-strain curves for single-edge-notched specimens with different scales subjected to direct tension and the simulated size effect results fitted by Bazant formula.

![Failure process](image2)

**Figure 4.6** Failure process of the sample subjected to tensile stress simulated by RFPA3D

![Crack propagation](image3)

**Figure 4.7** Propagation of three-dimensional crack in slices simulated by RFPA3D. Crack propagation paths are different on slices at different widths, which can not be simulated in 2D modeling.
The sequences of the single-edged crack growth for the largest specimen, being a representative of the five in the following descriptions, are shown in Figure 4.6. New crack initiated at the tip of the single-edged notch, and then it propagated ahead along a tortuous path perpendicular to the tensile stress. Due to the heterogeneity of the specimens, the crack path is not as smooth as the experimental results obtained in homogenous materials. The stress distribution is affected by the propagation of the crack. Note that there are a large number of fractured elements occurring when the crack reaches the upper boundary, which form a fracture zone much like FPZ (fracture process zone). Even under uniaxial tension, the crack propagation paths on different slices are not the same as depicted by Figure 4.7. It is obvious that such three-dimensional cracks can not be simplified into two-dimensional.

Bazant derived the following formula for geometrically similar structures (Bazant, 1984):

\[ \sigma_N = \frac{A}{(1 + \frac{\lambda}{B})^{1/2}} \]  \hspace{1cm} (4.1)

where \( A \) and \( B \) are positive coefficients. We can rewrite Eq. (2) and get a linear equation regarding the nominal strength and the characteristic size as follows:

\[ Y = a + b\lambda \]  \hspace{1cm} (4.2)

where \( Y = \left(\frac{1}{\sigma_N}\right)^2 \), \( a = \frac{1}{A^2} \) and \( b = \frac{1}{BA^2} \).

The nominal strength can be obtained by the following equation:

\[ \sigma_N = \frac{|f|}{A} \]  \hspace{1cm} (4.3)

Then we can derive the relation of the nominal strengths and the relevant \( \lambda \) from Eq. (4.2) and Eq. (4.3):

\[ Y = -0.2999\lambda + 2.9434 \]

From the numerical simulation results, size effect can be reliably accounted for in the numerical determination of the macro response of structures if the heterogeneous natures of the materials and crack propagation process, which may result in stress redistribution, are correctly modeled. It can be found that the nominal peak load decrease with the increase of the size of specimens. The computational nominal strength results can be fitted by the energy based formula proposed by Bazant’s (as shown in Figure 4.5).

**4.4 Mesh size sensitivity**

Numerical investigation of the mesh size sensitivity of the fracture model proposed in this thesis is given using carefully selected numerical experiments. The results confirm that it is necessary to employ relatively small elements in order to obtain good numerical results.

Six numerical specimens with the same mechanical parameters, including the mean value of the elastic modulus and the uniaxial compressive strength, Poisson ratio and residual strength coefficient, but different mesh network and element size are prepared to conduct uniaxial compression until failure. The side lengths of the cube element employed in the specimens are 1mm, 1.6mm, 2mm, 2.67mm, 4mm and 8mm, respectively.

The complete axial displacements versus stress curves as well as the fracture patterns are obtained
as shown in the Figure 4.8. It can be found that these six curves have little difference between each other in the elastic deformation stage due to few elements reach the stress criterion. All specimens undergo a soften stage with a residual strength. However, with the increase of the element size or the coarseness of the mesh, the stresses in the curves increase (Figure 4.9).

It can be found that the rock fracture model proposed is in general sensitive to element size. As can be found in Figure 4.10, if the coarse element (case A) is meshed into 16 elements (case D), according to the Weibull distribution function and the element discretisation way, the mean strength of the refined elements in case D is equal to the strength of the most coarse element. Therefore, the strength of the weakest element among these 16 elements will be smaller than the coarse element in case A and weakest element may reach its peak strength. In this way the crack would propagate through this zone if the more fine mesh is employed. With the decrease of the element size, the finer mesh will be generated, and the peak load of the specimens will decrease accordingly.

![Figure 4.8 Axial displacement versus stress curves of the six specimens with different mesh size.](image)

![Figure 4.9 Peak load versus mesh size of the six specimens.](image)

![Figure 4.10 Explanation of influence of mesh size on crack propagation in heterogeneous materials](image)

It can also be explained by the analysis of the stress distribution in the vicinity of the crack tip. For a very coarse mesh, the stress field close to the crack tip is mostly ignored by the finite element discretisation and the failure load obtained by such discretisation is close to the failure load obtained by simply assuming the uniform stress distribution. For a very fine mesh, the element size is in general only a fraction of the size of the fracture zone. Thus the fracture zone itself spreads through a relatively large number of finite elements and stress and strain fields within the fracture zone are well represented by the finite element approximation.

Numerical results presented in this thesis confirm that for very fine meshes accurate representation of stress and strain fields close to the crack tip and consequently accurate failure loads are obtained. Large-scale engineering problems where the size of the domain is measured in meters still require the smallest element to be of size as small as possible. However, in the case of highly fractured rock or rock masses mesh refinements in general result in very large finite element meshes and consequently in extensive CPU requirements. A suitable and proper algorithmic solution is a challenge for parallel computing, and the high performance computer is needed to meet the CPU and RAM requirement.
4.5 Direct tension fracture

Many numerical approaches are proposed to study the influence of the inhomogeneity in the rocks on the tension failure. However, due to the difficulty in three-dimensional modeling and high computational costs, most of these models have been generally developed in 2D. Five numerical specimens with different homogeneity indices were numerically tested, representing materials from relatively heterogeneous to homogeneity. In all cases, the specimens undergo uniaxial stress tension, imposed by a relative motion of the upper and lower ends with a constant rate of 0.0002 mm/step.

![Sketch map of the direct tension test](image1)

![Complete stress-displacement curves](image2)

![Fracture patterns of the five specimens with different homogeneity indices](image3)

Complete stress-displacement curves of the five specimens are obtained as shown in Figure 4.12. It is clear that the model predicts a non-linear load-displacement curve similar to the typical curve of disordered material observed in laboratory tests (Peng, 1975; Okubo and Fukui, 1996), and has essentially the same shape as compressive stress-strain curves. It is found that, although the constitutive law for individual element in the numerical model is brittle, a substantial nonlinearity exists before the maximum load, and the curve has an obvious post-peak region. Figure 4.13 shows the final fracture patterns of the five specimens subjected to uniaxial direct tension. It can be found that the fractures are not so smooth and flat, and there are no fractures occurring along the direction of the loading stress. Fractures are generally found at the end of the specimens in laboratory experiments due to the stress concentration resulting from the grip apparatus in direct tension test.

Crack propagation process as well as stress transition and redistribution can be reproduced in different loading steps by the numerical simulation as shown in Figure 4.15, in which the specimen has the homogeneity index $m=5.0$. Generally, the failure process of the specimen can be divided into three stages. Initially, randomly distributed microfractures are formed due to the heterogeneity of the
specimen. Once the crack nucleation initiated in some sites, they proceed relatively quickly to establish the nascent macrocracks but do not connect each other. Following this phase of macrocrack development is the growth phase, in which a band of microfractures develop, indicating a zone of intense stress concentration in the front area of the crack process zone (as shown in Figure 4.15).

Figure 4.14 Stress transition and redistribution in tensile fracture. The fractures are obviously 3D and they can not be simplified into 2D problems. The tensile stress will be redistributed with the crack propagation process.

Figure 4.15 Three-dimensional fracturing (failure plane) and AE distribution in the specimen (m=10.0) subjected to uniaxial tension. AE locations denote the fracture distribution.

4.6 Triaxial compression tests

Due to the difficulties in undertaking true triaxial compression tests, axisymmetric triaxial compression tests are always conducted in laboratory experiments even thought the intermediate principal stress effect can not be taken into consideration. Axisymmetric triaxial compressive tests under confining pressure been used as a simple and effective way to investigate the progressive failure process and associated microseismicities in rock material. Six axisymmetric triaxial compression tests are undertaken under different confining pressure (-2MPa, 0MPa, 2MPa, 5MPa, 10MPa, and 20MPa) to investigate the failure process of rock specimens and the confining pressure effect is discussed in terms of stress-strain curves, fracture patterns as well as peak strength. In the second part, another six true triaxial compression tests are undertaken under axisymmetric loading stress.

Figure 4.16 shows the confining pressure effect on the complete stress-strain curves and the peak strength. It can be found that the variation of the peak strengths of the specimens is markedly sensitive to the confining pressure applied. With the increase of the confining pressure, the peak strengths increase following a nearly linear function. In the post peak failure stage, the curves obtained for the six specimens have the similar shape. However, the residual strength of the
specimens cannot be independent of the confining pressure. The specimens with higher confining pressure applied have higher residual strength due to the enhanced constraint. The simulations reproduce the phenomena which have been approved by many researchers in axisymmetric compression tests.

Figure 4.17 gives the fracture patterns of the specimens under different lateral pressure. We observe that in all the tests specimens subjected to confining pressure have failed with the appearance of one or several shear bands through the specimen. It is found that the coalescence of cracked or weakened sites leads to the formation of eventual fracture planes or shear bands within a rock specimen under compression. In uniaxial compression, the collapse of a rock specimen is manifested by extensile fracture parallel to the applied stress appearing at the top of the specimen, and a combination of axial splitting and inclined failure surfaces is observed (Figure 4.17(a)). At moderate confining pressures, the eventual failure of the specimen is mainly characterized by one or more shear fracture planes (Figure 4.17 (b)(c)). At higher confining pressures, the appearance of intense deformation is exhibited by a ductile region, and the shear fracture plane becomes narrow compared with that under a low confinement (Figure 4.17(d)(e)). Compared with the laboratory results and the results simulated by Fang and Harrison (2002b), the numerical test in this thesis captures the essence of the features observed in laboratory tests and is consistent with the simulation performed by others though the rock type is different in the laboratory results.

![Figure 4.16 Confining pressure effect on the complete stress-strain curves and the peak strength](image1)

![Figure 4.17 Fracture modes of the specimens under different lateral pressure simulated by RFPA3D](image2)

According to the simulated fracture planes in the figures under different confining pressure, the orientation of the shear fracture planes to the loading axis increases with the confining pressure. Vertical and splitting fractures go through the specimen under uniaxial compression, and a sharp shear fracture zone forms in the moderate confined specimens. When the confining pressure increases to a value more than 30 MPa, the dip angle of the shear fracture to the principal stress reaches to the maximal value and nearly becomes horizontal in the specimen, where even no obvious
slip zone occur.

Axisymmetric confining pressure provides a useful way to carry out triaxial experiments to study the rock failure characteristics in three-dimensional stress state. However, the stress state of axisymmetric triaxial test is only a stress plane in 3D stress space, many experimental studies revealed that higher strength values were observed under plane strain loading and are believed to be due to the strengthening effect of the intermediate principal stress (Mogi, 1967 and 1979; Handin et al., 1967; Nascimento et al., 1974; Michelis, 1987; Yumlu and Ozbay, 1995).

Firstly, by keeping the minimal principal stress a constant value and varying the intermediate principal stress from 10MPa, 15MPa, 20MPa, 25MPa, 30MPa to 40MPa, six specimens are undertaken true triaxial compression tests and the peak strength as well as the compete stress-strain curves are simulated using RFPA3D code.

![Figure 4.18 Influence of intermediate principal stress on axial stress-strain curves and peak strength (σ₃=5MPa)](image)

As can be found in Figure 4.18, the complete axial stress-strain curves are influenced by the intermediate principal stress except the early elastic deformation stage, in which the all the stress are in proportion with the loading steps and there is no damage occurs. For the specimens under a moderate higher intermediate principal stress (15MPa and 20MPa), it takes more axial strain to reach the peak strength point. Different from the axisymmetric triaxial compression test, with the increase of the intermediate principal stress to a higher value compared with the moderate stress (30MPa and 40MPa), it is very interesting to find that the peak strengths of the specimens decrease even though the applied overall confining pressure increases. This means that with the increase of the intermediate principal stress, the strengths of the specimens will increase under a moderate intermediate principal stress, but will decrease under a high intermediate principal stress. The maximum peak strength will be achieved when the intermediate principal stress increases from the value of minimal principal stress to the value of maximal principal stress as can be shown in Figure 4.18.

Many researchers have observed the significant effect of intermediate principal stress on the peak
had two zones as described above by the numerical results. However, few literatures can be found to depict the fracture modes under different intermediate principal stress in true triaxial compression. Figure 4.19 shows these three different failure patterns of the specimens in the numerical tests influenced by the intermediate principal stress.

The effect of intermediate principal stress is also found to be similar in the other two groups of numerical studies by varying the minimal principal stress.

4.7 Three-dimensional fracture spacing in heterogeneous rocks

Spacing of opening-mode fractures (joints and veins) in a layered system is an important research topic in mechanical engineering, civil engineering and material sciences, as well as in the geosciences. Geoscientists have investigated fracture density in layered rocks because of its impact on the flow of groundwater and hydrocarbons, and the safety of mines (Price, 1966; Hobbs, 1967; Whittaker, 1990; Narr, et al., 1991; Gross, 1993; Wu and Pollard, 1995; Hong, 1997; Pollard, 1998; Bai, 2000).

Two types of fracture spacing are commonly observed—parallel fracture pattern and polygonal fracture pattern. Experimental and numerical studies with 2D embedded layer and layered half-space models have resulted in significant progress in the theoretical interpretation of the scale law of parallel fracture spacing with layer thickness (Bai et al., 2000), but seem insufficient to explain the observed complexity of real 3D polygonal fracture patterns. It was shown recently, however, that numerical simulation with 3D failure process approach and consideration of influence of heterogeneity on failure evolution allows the dynamics of pattern formation process of fracture spacing to be modeled directly. Here we present the numerical results of biaxial stretch tests on layered material models to investigate the influence of the heterogeneity of the material on the polygonal fracture pattern.

![Fracture patterns](image)

**Figure 4.20** Plots of simulated fracture patterns and minimal principal stress of polygonal fracture in different heterogeneous layered materials.

The two layer material models consist of 250×250×64 cells. For this two layered model, we postulate the two materials across the layer boundaries are welded together, i.e. no slip or opening is permitted along the layer boundaries (idea interface). We fix the whole bottom boundary in the vertical z-direction, and the top boundary is free to displace as necessary. A constant displacement increment, δd=0.0001mm, both in the x-direction and y-direction along the boundary is applied. Three numerical specimens with different homogeneity indices are subjected to biaxial stretch stress to generate polygonal fracture spacing until no more fractures form. The homogeneity indices of these specimens are 2, 3, and 10 respectively, representing three kinds of materials with various degrees of heterogeneity.

Figure 7.20(a-c) shows the plots of minimal stress of the three specimens with the homogeneity index m=2, 3, and 10 respectively. In the first stage of fracture pattern development, fracture nucleate at a small number of points in the surface layer. Defects (model cells with lower strength) in the layer
structure presumably serve as nucleation sites. In the homogeneous model these fractures then propagate in fairly straight lines. In run with inhomogeneous model, fractures tend to nucleate at the weaker sites and in many cases do not propagate long distances across the layer, but rather move in short steps from one weak site to the next, occasionally meeting another fracture moving in a similar fashion. For stronger disorder the cracks become wavy, but fragmentation still proceeds through crack growth. After the initiation of a few fractures, most new fractures start at the sides of existing fractures and propagate away from their parent fracture, approximately at right angles. Finally, successive generations of fractures form, mostly joining older fractures and forming a complicated array of polygons. In runs with homogeneous models, the faces of the fractures are smooth and the fractures, while they can curve smoothly, are usually locally straight, whereas in heterogeneous models, the patterns are more disordered in appearance.

![Crack patterns in heterogeneous materials simulated by T. Hornig et al. (a) fracture patterns in high heterogeneous material and (b) fracture patterns in relative homogeneous material.](image)

**4.8 Influence of heterogeneity on rock failure behaviors**

A series of numerical models, with the same elemental seed parameters but different homogeneous indices ($m = 1.1$, $m = 1.5$, $m = 2$, $m = 3$ and $m = 10$), are built to conduct uniaxial compressive strength tests. Figure 4.23 depicts the simulated stress-strain curves corresponding to the numerical specimens with the same mean values but different homogeneity. It can be seen that the stress-strain relation and the strength characterization depend strongly on the heterogeneity of the specimen. A similar phenomenon has also been observed in weak rocks (Tang et al., 2000a). Under uniaxial compression, lower homogeneity index values result in more heterogeneous characteristics in the stress-strain curve including a typical linear elastic deformation stage, a typical nonlinear deformation stage, a typical post-failure stage and a typical shearing and slipping stage (e.g. as shown in the cases of $m = 1.1$ and $1.5$). Correspondingly, the stress-loading displacement curve becomes more linear and brittle behaviour is more obvious. The numerically simulated results show that the mechanical processes of specimens with different heterogeneity features are more complex than the theoretical models, such as the fracture mechanics based on a single fracture description, and that the generalizations should not be made on the basis of limited results. In a relatively heterogeneous material, failure starts at a point where the stresses are not very high, which can be seen from curves in Figure 4.23 for the homogeneity indices of 1.1 and 1.5, where the nonlinear deformation region starts at a low stress level. It can be concluded that the sites where failure starts possess low local strengths due to pores, micro-fractures, grain boundaries, etc. in the simulated heterogeneous rock. In a relatively homogeneous material, failure initiates under higher stresses, which can be seen from the curves for the homogeneous indices of 3 and 10, where there is no obvious nonlinear deformation stage before the peak stress. Most of the failures are induced under the stress levels around the peak stress, which can be interpreted as a high stress failure. Moreover, even though the mean values of the mechanical parameters are all the same for these five specimens, it can be found that the peak strength of the specimens increases with the increasing of the homogeneity index. More homogeneous the specimen is, higher peak strength it has.
Variation or uncertainty of test results is an important feature in laboratory experiments. It is found in experiments that the localized zones or major fracture positions are different for different specimens due to the randomly distributed micro-defects in the specimen and they cannot be predicted (Hudson, 1993; Tang, 2000). The observations from our numerical simulations indicate that the macrofractures nucleated in different specimens with the same material parameters, suggesting that nucleation is strongly controlled by conditions of local variations of the mechanical properties.

![Complete stress-strain curves of numerical specimens with different homogeneity indices](image)

**Figure 4.23 Complete stress-strain curves of numerical specimens with different homogeneity indices**

![The variation of failure mode is sensitive to the local disorder (RFPA3D results). The mechanical properties for the four specimens are statistically the same on the macro-scale, the localized zones or major fracture positions are different from each other.](image)

**Figure 4.24** The variation of failure mode is sensitive to the local disorder (RFPA3D results). The mechanical properties for the four specimens are statistically the same on the macro-scale, the localized zones or major fracture positions are different from each other.

![Complete stress-strain curves of four specimens with the same homogeneity index. The macro mechanical responses of the specimens are more or less the same to each other. Local disorder has little influence on the macro mechanical behavior.](image)

**Figure 4.25 Complete stress-strain curves of four specimens with the same homogeneity index. The macro mechanical responses of the specimens are more or less the same to each other. Local disorder has little influence on the macro mechanical behavior.**

The simulations were conducted with four specimens with the same seed parameters ($m = 2.0$ for all five specimens) as that used in the above study. Since the computer generates the mechanical parameters of the elements randomly by following the Weibull's distribution, the four specimens with the same macro properties will have different local characteristics for individual elements. While all specimens appear to behave similarly during the initial loading stage, considerable differences can be observed in the post-failure region. Figure 4.24 shows the results of the final
failure modes of the four specimens. The results indicate that variation of failure mode is strongly sensitive to the local disorder feature of the specimen as soon as the microfracture nucleated. Although the mechanical properties and the initial stress distribution for the four specimens are statistically the same on the macro-scale, the localized zones or major fracture positions for the four specimens are different from each other. Consequently, the final failure modes are different. It is worthy of noting that, as shown in Figure 4.25, although the failure modes differ considerably, the stress-strain curves for the four specimens demonstrate a more or less identical shape.

5 Discussions

5.1 Fractal characteristic of three-dimensional fracture

Five specimens with different homogeneity indices were numerically tested aimed to study the tensile fracture and associated fractal characteristics, representing materials from relatively heterogeneous to homogeneity. The diagrammatic sketch of direct uniaxial tension test is shown in Figure 4.5. The fractal dimensions for all the specimens has a sharp increase in the non-linear deformation stage and keeps a constant when the final fracture surface is formed. Figure 5.1 plots the evolution of AE location throughout the specimen with m=6.0 as well as the fractal dimension during the failure process. The dispersed acoustic emission distribute throughout the entire specimen in the first stage, and with the propagation of the pre-existing notch, the AE events gather round in the vicinity of the fracture surface. Figure 5.1 shows the final fracture figures of the notched specimens with the homogeneity index m=1.5, m=6.0, m=15.0. It is obvious that the fracture surface is much coarse for the most heterogeneous specimen m=1.5, and the surface is most smooth and flat for the relatively most homogeneous specimen m=10.0. The fractal dimensions of the AE were determined by analyzing the distribution of the failure elements in the specimens, which scattered along the fracture surface. The fractal dimension is close to 2.0 for the specimen m=15.0, and it shows that the failure elements almost distribute on the fracture surface, while for the specimen with m=1.5, the fractal dimension is about 3.0, and this means that the failure elements distributes almost throughout the entire specimen.

![Figure 5.1](image.png)

Figure 5.1 Evolution of AE locations throughout the specimen with m=6.0 as well as the fractal dimension variation during the failure process. AE events cluster in the vicinity of the prepared notch. The fractal dimension increases gradually and reaches 2.5 approximately at the final rupture.

The failure behavior shows more ductile and a lower average peak tensile strength for the relatively more heterogeneous specimens, while the relatively more homogenous specimens fracture
abruptly and a low residual stress is found. The fractal results show that the fracture surface is more smooth and flat for the relatively more homogeneous specimens. As a result, a smaller fractal dimension is obtained.

![Figure 5.2 Final fractures for the notched specimens with different homogeneity indices under tension (RFPA^3D results).](image)

**Figure 5.2** Final fractures for the notched specimens with different homogeneity indices under tension (RFPA^3D results).

![Figure 5.3 (a) Fractal dimensions versus loading displacement for different heterogeneous rock specimens. The fractal dimension increases with the loading process. More heterogeneous specimens give a higher fractal dimension. (b) Relation of final fractal dimension to homogeneity index. The fractal dimension decreases with the increasing of the homogeneity index.](image)

**Figure 5.3** (a) Fractal dimensions versus loading displacement for different heterogeneous rock specimens. The fractal dimension increases with the loading process. More heterogeneous specimens give a higher fractal dimension. (b) Relation of final fractal dimension to homogeneity index. The fractal dimension decreases with the increasing of the homogeneity index.

It is interesting to find that the fractal dimension of AE decreases as the index of the homogeneity increases. From the numerical simulation, the fractal dimension as well as homogeneity index can be regarded as a significant parameter to define the brittle or ductile degree of different rocks.

### 5.2 Acoustic emission modes

According to the homogenous degree, three basic modes of seismic activities are found: swarm shocks, pre-main-after shocks, and main shock. Figure 5.4 shows the sequences of acoustic emission of the specimens with \( m = 1.1, 3, \) and 10 subjected to uniaxial compression until failure. Swarm shock can be found in much heterogeneous rocks. Microfractures and AE scattered here and there throughout the specimen and acoustic emission could be detected at initial stage. Pre-main-after shocks could be found in more homogenous rocks and both acoustic emissions can be detected before and after the main macrofractures was formed (Figure 5.4). Main shock could be found in most homogenous rocks. In more homogenous rocks, it is hard to predict the precursors because they showed only a small number of acoustic emissions. The results showed that relatively heterogeneous specimens emitted more acoustic emission as precursors of macrofractures than that of relatively homogenous specimens at the first loading stage. A greatly larger number of acoustic emissions were recorded in homogenous numerical rock specimens than in heterogeneous ones when the specimen reached its peak strength.

Three dimensional numerical results showed good agreement with two dimensional numerical results (Tang, 2000), laboratory experimental results and three basic earthquake modes observed by many researchers (Mogi, 1985).
5.4 Simulated modes of seismic activities in rock failure process

(a) swarm shocks  (b) pre-main-after shocks  (c) main shock

5.5 Acoustic emission and associated fractal dimension evolution in rock fracture process $(m=3.0)$. The fractal dimension $D$ of the acoustic emission distribution during loading is measured by box counting method. The AE gathers in the vicinity of the fracture and the $D$ increases to a nearly constant value when the specimen collapses. The balls in the AE pictures represent AE events, and the radius of the balls represents the relative energy release at each loading step.

5.3 Intermediate principal stress effect

By changing the minimal principal stress, another two groups of numerical tests are conducted in the same manner as described in 4.6. It can be found that the significant effect of intermediate principal stress on the peak had two zones: (1) rock strength $(\sigma_1')$ increases with increasing the intermediate principal stress $(\sigma_2)$ from $\sigma_2 = \sigma_1$ to a maximum value (reach the peak strength when $\sigma_2 = \sigma_2'$); (2) rock strength $(\sigma_1)$ decreases with further increasing of the intermediate principal stress $\sigma_2$ from $\sigma_2 = \sigma_2'$ to $\sigma_2 = \sigma_1$.

The failure criterion may explain the intermediate principal stress successfully. It seems that the conventional failure criterions, including Mohr-Coulomb criterion and Hoek-Brown criterion, which
do not include the intermediate principal stress effect at all, and Drucker-Prager criterion, which includes this effect but equates it to the effect of the maximum principal stress and the minimum principal stress, can not interpret this effect soundly. USC (Unified Strength Criterion) is included in RFPA\textsuperscript{30} code, and TSC (Twins Shear Criterion), an extreme form of USC is treated as the default criterion. An advantage of the TSC is that the intermediate principal stress is considered.

The intermediate principal stress effect can be explained with the Twins Shear Criterion. When the intermediate principal stress increases from $\sigma_2 = \sigma_3$ to a certain value $\sigma_2' = \frac{\sigma_3 + \alpha \sigma_1}{1 + \alpha}$, the strength criterion function can be described as follows:

$$F = \alpha \sigma_1 - \frac{1}{2}(\sigma_2 + \sigma_3) - \sigma_i$$

$F$ is a monotonic decreasing function with the increasing of $\sigma_2$. This means that the increasing of $\sigma_2$ will result in fewer elemental failure within the numerical model. It is obvious that the strength of the specimen will be enhanced when $\sigma_2$ increases in this zone.

However, by further increasing the intermediate principal stress from $\sigma_2' = \frac{\sigma_3 + \alpha \sigma_1}{1 + \alpha}$ to $\sigma_i$, the strength criterion function can be described as follows:

$$F' = \frac{\alpha}{2}(\sigma_1 + \sigma_2) - \alpha \sigma_1 - \sigma_i$$

Here $F'$ is a monotonic decreasing function with the increasing of $\sigma_2$. On the contrary, the increasing of $\sigma_2$ will result in more elemental failure, which will lead to a smaller strength of the specimen.

In addition, there is no doubt that the critical value $\sigma_2'$ will increase with the increase of and it is interesting to find that the numerically obtained critical value increases from around 5 MPa to 25 MPa when the minimal principal stress $\sigma_3$ increase from 0MPa to 10MPa. The numerical result is consistent with the theoretical analysis.

In Michelis's investigations (1987), he found that the intermediate principal stress obviously has influence on the elastic modulus, the cohesion and the friction angle except the peak strength of the specimens. From the numerical results, it can be found that it has the same effect on the elastic modulus as on the rock strength. The macro response of the specimens shows that the elastic modulus increases in one stage when $\sigma_2$ increases from $\sigma_3$ to $\sigma_2'$, while it decrease in the other stage when $\sigma_2$ increases from $\sigma_2'$ to $\sigma_i$. 

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The intermediate principal stress also influences the fracture pattern significantly, which has been ignored by many researchers when investigating the intermediate principal stress effect. The surface of the slipping fracture occurring parallel to the intermediate principal stress moves along the orientation of minimal principal stress as shown in Figure 5.6. According to the value of the intermediate principal stress relative to the maximal principal stress and the minimal principal stress, there are three fracture patterns can be observed. When the intermediate principal stress is as small as the minimal principal stress, the specimen presents a failure pattern as can be found in low confining compression or in uniaxial compression (as shown in Figure 5.6(a)). When the intermediate principal stress increases to a higher value, the orientation between the fracture plan and the maximal principal stress decreases and it is often found that the one or more fractures will go though the top or the bottom of the specimens (as shown in Figure 5.6(b)(c)). When the intermediate principal stress increases further to a value close to the maximal principal stress, many fractures parallel to both the maximal principal stress and the intermediate principal stress and perpendicular to the minimal principal stress (as shown in Figure 5.6(d)). The intermediate principal stress effect
on the fracture patterns may result from the constraint of the intermediate principal stress applied on the specimen relative to the maximal principal stress and minimal principal stress.

5.4 Pattern transition from parallel to polygonal fractures

![Image of fracture patterns]

Figure 5.8 Plots of the minimal principal stress distribution of the specimens subjected to tensile stress.

Polygonal fractures have been observed and analyzed when the layered rocks are subjected to isotropic tension stresses. However, since the directional tension will result in an anisotropic stretch in the tested layer, we believe that the laddering fracture pattern should relate to another loading condition, that is, between the isotropic stretch and the directional or uniaxial stretch. It is believed that transition of fracture pattern from parallel to polygonal fractures in layered rocks depends strongly on the far-field loading conditions in terms of principal stress ratio \((\lambda = \sigma_2 / \sigma_1)\), from uniaxial (\(\lambda = 0\)), biaxial to isotropic stress (\(\lambda = 1\)).

In the numerical investigations, three types of numerical tests were performed: “isotropic stretch,” in which the entire layer was stretched uniformly with principal stress ratio \(\lambda = 1\); “anisotropic stretch,” in which the layer was stretched biaxially with principal stress ratio \(0 < \lambda < 1\) and “directional stretch,” in which the layer was loaded from two ends, that is, uniaxially with principal stress ratio \(\lambda = 0\). We observed a change in the way in which fracture occurred as the loading method in the system was varied: for isotropic stretch, more regular shaped polygons formed. For anisotropic loading, polygons did not formed in regular shape, but rather formed by laddering. In this process, fractures propagate more or less parallel to each other. As the stretch strain increased further, perpendicular fractures form, joining two of the parallel fractures like the rungs of a ladder. The uniaxial stretch produces the pattern of parallel fractures. The transition from parallel to polygonal fractures under different loading conditions is shown in Figure 5.8.

In addition, in three-dimensional modeling, the interface debonding is found to be dominating the fracture development upon reaching the fracture saturation stage, which is a big effect in laboratory experiments and is clearly not considered in the 2D models of the time (as shown in Figure 5.9). The previous theory about fracture spacing using two-dimensional and plane-strain models suggests that further strain after the fracture saturation will be accommodated by the opening of the existing fractures. Our three-dimensional modeling provides a second mechanism for this strain accommodation, that is, the fractures propagation along the layer interface.

It can be found in the fracture patterns that the number of the parallel fractures in one direction is in proportion to the value of the value of the corresponding stress applied in this direction. It seems that the stress in the one direction has little influence on the fracture formation in the other direction. Figure 5.10(a) shows the distribution of the stress of a numerical specimen with a pre-existing crack subjected to isotropic tension. Stress concentration is found at the tip of the cracks, which will lead the propagation of the pre-existing crack. As shown in Figure 5.10(b), the stress in X direction (on the
line a-a') decreases greatly shielded by the pre-existing crack oriented to X direction. However, the pre-existing crack has little influence on the stress distribution in Y direction (on the line a-a'). This means that the fractures formed in X direction will prevent the initiation of the new fractures oriented to X direction, but fractures in X direction can transfer stress in Y direction and it has little influence on the formation of fracture formation in Y direction. Perhaps the anisotropic feature of the fractures is the main reason of the transition of patterns from parallel to polygonal fractures depending on the stress ratio of loading conditions.

![Figure 5.9 Fracture patterns under isotropic tension in different stages. The interface debonding is found to be dominating the fracture development upon reaching the fracture saturation stage, which is a big effect in laboratory experiments and is clearly not considered in the 2D models of the time.](image)

![Figure 5.10 Stress fields near the crack subjected to isotropic stress. Anisotropic feature of the fractures is the main reason of the transition of patterns from parallel to polygonal fractures](image)

**5.5 Transitions of brittle to ductile failure**

Even though the constitutive law for the mesoscopic element is almost brittle, the macro response of the specimens may show a ductile or semi-ductile mechanical behavior as can be found in the previous numerical tests. As stated by Besuelle, failure in rock is generally characterized by a brittle regime, a transitional semi-brittle regime and a ductile regime (Besuelle et al., 2000). It can be found from the numerical investigation that the mechanical behavior of the specimen is greatly affected by the confining pressure as well as the heterogeneity, as can be shown in Figure 4.16 and Figure 4.23 (for the specimen m=1.1) respectively.

The discussion of the confining effect on the strength and the failure behaviors has been presented in Figure 4.16. Moreover, the non-linear mechanical behavior is influenced by the confining pressure significantly. The deformation and failure are brittle, if the confining pressure is zero or relatively low. Under uniaxial compression, the fracture processes develop very quickly, so that the specimen collapses over a very small strain range, which is the brittle regime. Dilation under a stress higher than the yield strength and a post-failure stage with a descending load bearing capacity are the
prominent characteristics in this case. At the post-failure stage, the stress-displacement curve of the sample experiences stress drops several times before the axial stress reaches a residual level, and the behaviour of the material is brittle. As the confining pressure increases, the strain hardening range increases and both the strength loss after the peak and the brittleness of the curve decrease. Finally, when the confining pressure increases further, the material behaviour becomes plastic.

It is well known that the confining pressure plays an important role on the transitions of brittle to ductile failure in rocks as can be observed in many experimental studies. However, many researchers have ignored that heterogeneity also has the influence on this transitions.

It can be found in Figure 4.23, for the more heterogeneous rock specimens ($m=1.1$ and $m=1.5$), the stress-strain curves are characterized by non-linear deformation before the peak strength, and they demonstrate very gentle post-peak mechanical behavior. As a result, the strength loss, even more than zero, is very little, and the specimens show an obvious ductile deformation in the post-peak region. However, for the more homogeneous rock specimens ($m=3.0$ and $m=10.0$), the stress-strain curves show nearly a linear elastic deformation before the peak strength, and the strength loss is also sharper, which leads to an obvious brittle failure. Even though the specimen with homogeneity index smaller than 1.1 has not been analyzed, it can be predicted that a more ductile failure behavior can be observed for more heterogeneous rocks.

### 5.6 Failure criterion and multi-axial tests

In addition to twins shear failure criterion, Mohr-Coulomb failure criterion and Drucker-Prager failure criterion are applied to judge the failure of the elements in the model subjected to uniaxial compression with different confining pressure at each step.

Even though the applied intermediate principal stress and the minimal principal stress are zero, the internal intermediate principal stress and the minimal principal stress of the model can be larger or smaller than zero due to the heterogeneous properties in different elements. The numerical results show that there is a little difference between the complete stress-strain curves as well as the peak strength obtained by applying different failure criterions under uniaxial compression. The heterogeneous stress distribution may lead to stress localization and failure localization, which will lead to further heterogeneous stress distribution.

The strength of the models increases with the confining pressure by applying different failure criterions. Strength increment obtained by using twins shear failure criterion is greater than Mohr-Coulomb failure criterion, and the increment obtained by using Mohr-Coulomb failure criterion is greater than Drucker-Prager failure criterion. According to the description of the failure criterions, the yield surface of the Mohr-Coulomb failure criterion is enveloped by the yield surface of the Drucker-Prager failure criterion used in our studies, and the yield surface of the Drucker-Prager failure criterion used in our studies is enveloped by the yield surface of the twins shear failure criterion.

It is interesting to find that the confining pressure has little influence on the peak strength by using Drucker-Prager failure criterion, even though the strength increases more or less when the confining pressure increases. This maybe results from the fact that the hydrostatic pressure is considered in Drucker-Prager failure criterion. Even under low compressive loading, many elements fail in the beginning loading stage due to the hydrostatic pressure in three-dimensional stress. This phenomenon should be investigated in the further studies.

The intermediate principal stress is overestimated in the twins shear failure criterion. Even though it is difficult to determine the value of the parameter $b$ in the United Strength Criterion, it has provided a better choice to consider the effect of intermediate principal stress under multi-axial
loading condition.

5.7 Distribution functions

The mechanical behaviors obtained in the heterogeneous numerical model implemented by using both even distribution function and normal distribution function are similar to the Weibull distribution function. The non-linear behavior and the brittle failure become more obvious in more homogeneous models, and the peak strength increases with the homogeneity.

In even distribution function, the homogeneity index can be described by the ratio of \((b-a)\) or \((b-a)^2\) to the expectation value \((b+a)/2\). However, it is difficult to determine either the value of \(a\) or \(b\). In addition, the meaning of the homogeneity index is ambiguous. Normal distribution function is widely used in statistical method and the homogeneity index can be described by the ratio of the variance \(s\) to the expectation value \(E\). Logarithmic normal distribution has the same disadvantage which will restrict their application in describing the heterogeneity in rocks. Probably Weibull distribution function is the most simple and convenient one among these distribution functions relatively.

6 Conclusions

In this thesis, a novel numerical model is proposed on the basis of the mesoscopic elastic damage model and the parallel computing method. Three-dimensional rock failure progressive process in various mechanical conditions is investigated by conducting a serial of numerical tests with the aid of a numerical code RFPA\textsuperscript{3D}. The following conclusions can be drawn from the numerical studies described.

(1) Many statistical distribution functions, such as Weibull distribution function, even distribution function and normal distribution function, are introduced into the numerical model to describe the mechanical heterogeneous properties. Disorder can be implemented by means of randomly distributions of the mechanical properties of the elements in the numerical model.

(2) In addition to some traditional strength criterions, the modified Unified Strength Criterion with tension cut-off is introduced into the model to consider the intermediate principal stress effect on rock shear failure in three-dimensional stress. According to different strength criterions the elements satisfy, the elemental failure can be regarded as either in shear failure or tensile failure.

(3) The evolution of the mechanical properties of rock under mechanical loading can be described using the mesoscopic elemental mechanical model for elastic damage. In the model, when the stress state of an element meets the damage threshold, the element will be damaged in the tensile or shear mode. The main features of the proposed mesoscopic mechanical model are that there is no need for pre-fabricated flaws to simulate the failure initiation or fracture propagation.

(4) There are four modules to implement the RFPA\textsuperscript{3D} code: preprocessing module, FEM module, failure analysis module and postprocessing module. The preprocessing module, failure analysis module and postprocessing module are developed by Microsoft Visual C++ on Microsoft Windows XP operation system, while the FEM module is developed by MPICH on Redhat9.0 operation system. FEM module for PC is developed with FORTRAN 95 language only, while FEM module for cluster computers is developed with FORTRAN 95 language and MPI (Message Passing Interface) on the LINUX operation system. The numerical results show that for the analysis of the three-dimensional irregular and complicated rock fractures, the parallel computing code is effective and reliable. Moreover, regardless that the scale of computing degree of freedom or the number of processes (CPUs), the parallel program don’t need to be modified. This shows that the parallel computing performance of the FEM module is stable and extensible.

(5) Numerical tests for the typical mechanical experiments indicate the developed RFPA\textsuperscript{3D} code is a
valuable numerical tool for research on the rock failure progressive fracture by comparing with the results of theoretical analysis, other numerical studies, laboratory experimental studies as well as RFPA2D code.

(6) The simulations for different platen conditions show that the type of fracture is markedly sensitive to the platen or end conditions. The crack patterns show that almost-vertical splitting cracks develop in specimens loaded with softer loading platens, and the hour-glass failure mode develops in specimens loaded with stiffer loading platens. In addition, the results indicate that a more ductile response is simulated and the peak strength increases with the increasing of the constraint.

(7) The 3D numerical simulations show that peak stress depends strongly on the ratio of the specimen height to width / thickness. The peak stress sustained by a specimen decreases with increasing slenderness of the specimen. The pre-peak portion of the stress-strain curves shows no significant dependence on slenderness; however, the post-peak curves are highly dependent on the ratio of specimen height to width. Due to the friction between the loading platens and the specimen, rock specimens have a complicated triaxial stress state in the end zones. The numerical results suggest that by changing the ratio of height to diameter in laboratory experiments the contact effect can be eliminated.

(8) Size effect can be reliably accounted for in the numerical determination of the macro response of structures if the heterogeneous natures of the materials and crack propagation process, which may result in stress redistribution, are correctly modeled. It can be found that the nominal peak load decrease with the increase of the size of specimens. The computational nominal strength results can be fitted by the energy based formula proposed by Bazant's.

(9) With the increase of the element size or the coarseness of the mesh, the load increases. For a very fine mesh, the element size is in general only a fraction of the size of the fracture zone. Thus the fracture zone itself spreads through a relatively large number of finite elements and stress and strain fields within the fracture zone are well represented by the finite element approximation.

(10) Numerical tests suggest the orientation of the shear fracture planes to the loading axis increases with the increasing of the axisymmetric confinement. The specimens with higher confining pressure applied have higher residual strength due to the enhanced constraint. The simulations reproduce the phenomena which have been approved by many researchers in axisymmetric compression tests.

(11) The three-dimensional evolution of the polygonal fracture depends on the heterogeneity of the layered material under isotropic stretch. In much heterogeneous materials the formation of the polygonal patterns depend more on crack initiation than on crack propagation. For the much heterogeneous the rocks, the polygonal fractures become more wavy and irregular. On the contrary, the polygonal fractures are formed in fairly straight lines regularly for the relatively homogeneous layered rocks.

Three-dimensional simulations show the transition of fracture pattern from parallel to polygonal fractures in layered rocks depends strongly on the far-field loading conditions in terms of principal stress ratio \( \lambda = \sigma_2 / \sigma_1 \), from uniaxial\( \lambda = 0 \), biaxial to isotropic stress \( \lambda = 1 \). For isotropic stretch, more regular shaped polygons forms. For anisotropic loading, polygons do not form in regular shape, but rather form by laddering, and the uniaxial stretch produces the pattern of parallel fractures. The anisotropic feature of the fractures is the main reason of the transition of patterns from parallel to polygonal fractures depending on the stress ratio of
loading conditions.

(12) The fractal dimensions for all the pre-notched specimens subjected to uniaxial tension has a sharp increase in the non-linear deformation stage and keeps a constant when the final fracture surface is formed. The fractal dimension of AE decreases as the index of the homogeneity increases. From the numerical simulation, the fractal dimension as well as homogeneity index can be regarded as a significant parameter to define the brittle or ductile degree of different rocks.

(13) Three basic seismic activity modes can be found in different kinds of rocks: swarm shock can be found in much too heterogeneous rocks, for which it is difficult to determine the main shock among all seismic activities, pre-main-after shocks could be found in relative homogenous rocks, for which both acoustic emissions can be detected before and after the main macrofractures, and main shock could be found in most homogenous rocks, for which it is hard to predict the precursors because they showed only a small number of acoustic emission.

(14) The intermediate principal stress effect can be reproduced by RFPA3D in true triaxial compression tests. Numerical results show that the significant effect of intermediate principal stress on the peak had two zones: (1) rock strength ($\sigma_1$) increases with increasing the intermediate principal stress ($\sigma_2$) from $\sigma_2 = \sigma_3$ to a maximum value (reach the peak strength when $\sigma_2 = \sigma_1$); (2) rock strength ($\sigma_1$) decreases with further increasing of the intermediate principal stress $\sigma_2$ from $\sigma_2 = \sigma_3$ to $\sigma_2 = \sigma_1$. The intermediate principal stress also influences the fracture pattern significantly, which has been ignored by many researchers when investigating the intermediate principal stress effect. The intermediate principal stress effect on the strength of rocks can be explained with the Twins Shear Criterion or Unified Strength Criterion.

(15) Heterogeneity in rock-like materials is one of the most important factors that influence the mechanical behavior of materials. Even though the mean values of the mechanical parameters are all the same for these five specimens with different heterogeneous properties, the peak strength increases with the increasing of the homogeneity index. However, the results indicate that variation of failure mode is strongly sensitive to the local disorder feature of the specimen as soon as the microfracture nucleated. Although the mechanical properties and the initial stress distribution for the five specimens are statistically the same on the macro-scale, the localized zones, major fracture positions and fracture modes for the five specimens are different from each other.

(16) Compared with the normal distribution function, even distribution function and logarithmic normal distribution, probably Weibull distribution function is the most simple and convenient to describe the heterogeneity in rocks.

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