THE INFLUENCE OF ROCK ANISOTROPY ON MEASUREMENT OF STRESSES IN SITU

Bernard Amadei

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Signature Richard E. Goodman
Richard E. Goodman
Thesis Committee Chairman

ABSTRACT

A medium is anisotropic if its properties vary with direction. This is the general characteristic of many rocks, for examples, schists, slates, gneisses, phyllites and other metamorphic rocks. Bedded and regularly jointed rocks also display anisotropic behavior.

The purpose of this dissertation is to evaluate the influence of rock anisotropy on in-situ stress measurements. The thesis is limited to stress measurements by overcoring techniques for which strains and displacements are recorded on the walls of a borehole or within instrumented hollow or solid inclusions perfectly bonded to the surface of that borehole. The rock is described as homogeneous, continuous, anisotropic, and linearly elastic.

The constitutive relations and failure criteria available for anisotropic media are reviewed and their applicability to rocks is analyzed. Laboratory and in-situ techniques used to assess the directional character of the deformability and strength of aniso-

tropic rocks are also discussed.

Closed form solutions are obtained to connect strain and displacement components at any point along the walls of a circular hole, or within a hollow or solid inclusion perfectly bonded to that hole and a 3D stress field applied at infinity. No restrictions are made on the type or orientation of the anisotropy or on the orientation of the applied stress field with respect to the hole.

These closed form solutions are applied to calculate 3D in-situ stress field components from measurements of strains and displacements associated with existing overcoring techniques in one or several boreholes. New techniques are also proposed involving low-modulus, thin-walled hollow inclusions or very-low-modulus, solid inclusions.

The following questions are answered with special emphasis to rocks that can be classed as transversely isotropic, or orthotropic: the number of independent measurements obtainable in a single borehole; the number of boreholes required to determine the in-situ stress field; the influence or rock anisotropy on these numbers; the influence of the anisotropy type and orientation on the determination of the in-situ stress field, and the error involved by neglecting rock anisotropy.

Chapter 7: Summary and Conclusions

In order to assess the influence of rock anisotropy on in situ stress measurements from a quantitative point of view, knowledge of the directional character of the rock deformability is required. In this study, the latter is described using the constitutive relations of the theory of linear elasticity for anisotropic media. Directions of elastic symmetry are assumed to coincide with the apparent directions of rock anisotropy. The extent to which this assumption is applicable must always be investigated by testing rock specimens in the laboratory and in situ. Procedures are proposed in Chapter 2 to calculate the five and nine elastic constants of rocks that can be described respectively as transversely isotropic or orthotropic materials using unconfined, triaxial or multiaxial compression laboratory tests. If more than nine elastic constants are needed, the procedure becomes more complex but may be investigated as a possible model to describe the deformability of anisotropic rocks that present directions of anisotropy inclined with respect to each other.

The elastic properties measured using laboratory tests may not be representative of the deformability of the rock in place since scale effects are important in rock mechanics. In situ tests reported in the literature for the determination of the elastic constants of anisotropic rock masses are not numerous. Additional assumptions are often stated in order to reduce the number of elastic constants that must be known. A procedure is proposed here to calculate the elastic properties of anisotropic rock masses that can be described as orthotropic or transversely isotropic by means of a dilatometer test. Since there are at most two independent measurements of diameter change obtainable from the expansion of a circular hole, general determination of the five of nine elastic constants is impossible from measurements in a single borehole. Therefore several non parallel boreholes must be drilled within the region of interest or additional assumptions are required regarding the elastic constants. However, when the anisotropy is derived from a regular joint set striking parallel to a borehole and the intact rock is isotropic, the two changes in diameter are sufficient to determine the normal and shear stiffnesses of the joint set as long as its spacing is known and the stiffnesses are assumed constant. In theory, the elastic modulus of the intact rock can be determined by pressurizing and unpressurizing the hole if the joint deformations are assumed to be non recoverable and the rock to be elastic. This procedure can be applied also if the rock is cut by three orthogonal joint sets having identical properties. This leads to a possible method to evaluate the modulus of deformation of a regularly jointed rock mass.

A prerequisite for the applicability of stress measurement techniques is that the strength of the rock is never exceeded anywhere around the hole where measurements take place. This is particularly important if the stresses to be measured are expected to be high. From a practical point of view, rock failure can occur as indicated by core discing or flaking of borehole walls. Obviously, measurements obtained from a borehole where those phenomena take place should not be interpreted using the theory of linear elasticity. Chapter 3 deals with the strength of anisotropic rocks but not exhaustively. This chapter simply reviews the experimental evidence for the directional character of the strength of anisotropic rocks and the analytical models available to account for this property. It is suggested that an appropriate failure criterion for anisotropic rocks must be sufficiently general to include the influence of the intermediate principal stress and to account for the type and orientation of the anisotropy. The general form of the failure criterion is patterned after that for the deformability. It depends on the results of laboratory tests and the apparent symmetries of the rock material.

The tensile strength of anisotropic rocks can be calculated using direct or indirect methods (e.g. the "Brazilian" test). Interpretation of test results must include the anisotropic character of the rock. In particular, if a disc of rock is loaded with two line loads, the stress distributions within the disc calculated from the theory of linear elasticity must include the anisotropic character of the rock, its symmetry if any and the orientation of the anisotropy with respect to the direction of loading. It is surprising that common practice uses the isotropic solution to calculate the tensile strength of anisotropic rocks even though an anisotropic solution is available (Okubo, 1952).

General closed form solutions are proposed in Chapter 4 for the distributions of stresses, strains and displacements at any point within an infinite linear elastic, anisotropic, continuous and homogeneous body bounded internally by a cylindrical surface of arbitrary cross section. The body is subjected to body forces and boundary stresses acting along its internal contour. The work of Lekhnitskii (1963) used as a guide line, is extended to include the influence of a boundary stress component parallel to the hole axis. The anisotropy is general and described by 21 independent elastic constants in the coordinate system attached to the hole.

From a practical point of view, these closed form solutions are complicated and simplifying assumptions are often required. The different formulations of plane strain and plane stress available in the literature are reviewed and their applicability to anisotropic media discussed. In particular, it is found that the assumptions associated with the usual plane strain and plane stress formulations imply undue restrictions on the orientation of the hole with respect to the anisotropy, the mode of loading and the mode of deformation of the anisotropic body. Instead, the generalized plane strain formulation introduced by Lekhnitskii (1963) and Milne-Thomson (1962) can be used for any mode of loading and orientation of the anisotropy. The only assumption is that stresses, strains and displacements induced by the application of the boundary stresses be the same in any cross section perpendicular to the longitudinal axis of the hole. The approximate character of the existing plane stress formulations is also emphasized. Furthermore, all of them require that the hole be perpendicular to a plane of elastic symmetry.

As a special case of the general theory, a closed form solution is derived between the components of stress, strain and displacement at any point within an infinite anisotropic body bounded internally by an infinite cylinder with a circular cross section and the components of a 3D stress field applied at infinity. The hole axis, the anisotropy and the 3D applied stress field are inclined with respect to each other. The model allows for a constant longitudinal strain.

This strain is induced by the application of the original 3D stress field on the anisotropic medium lacking a hole even though the process of drilling the hole induces zero longitudinal strain. Stresses, strains and displacements induced by the hole drilling can be calculated using the generalized plane strain formulation. The latter reduces to the classical plane strain formulation if the hole is perpendicular to a plane of elastic symmetry and is drilled in a principal direction of the applied stress field.

The general theory can be used also for other cases of boundary stresses such as the application of an internal pressure uniformly distributed along the contour of a circular hole or more complex boundary stress distributions such as those associated with borehole jacks. This should allow anisotropy to be included in the analytical formulations for the Goodman jack in association with the measurement of deformability or strength of anisotropic rock masses.

Chapter 5 analyzes the elastic equilibrium of an infinite, continuous, homogeneous and anisotropic body bounded internally by an isotropic hollow or solid inclusion of circular cross section. The inclusion is assumed to be infinitely long and perfectly bonded to the anisotropic body. Closed form solutions are developed, in matrix form, between the total components of stress, strain and displacement at any point within the inclusion located far from its ends and the components of a 3D stress field applied at infinity. The model allows for a constant longitudinal strain within the anisotropic body and the inclusion. A general expression is also derived to relate the change in length between two points located in different cross sections of the inclusion to the components of the 3D applied stress field.

Another point of interest in Chapter 5 is the influence of the value of the elastic modulus of the inclusion and its geometry upon the amount of disturbance of the stress field associated with the presence of the inclusion within the anisotropic body. Numerical examples are presented for isotropic and transversely isotropic media containing a high or a low modulus inclusion. For these examples, an inclusion is arbitrarily defined as having a high modulus if the ratios of the Young's modulus of the inclusion to the Young's moduli of the anisotropic body are all larger than 0.5. It is defined as a low modulus inclusion if these ratios are all less than 0.1. The following conclusions can be drawn:

a) at any point along the external contour or within a hollow inclusion, the principal stress field becomes more uniform as the inclusion becomes thicker. It is completely uniform for a solid inclusion as predicted by Eshelby (1957), its magnitude being mostly affected by the high or low modulus character of the inclusion. Its orientation seems to be little influenced by the orientation of the anisotropy and is close to the principal applied stress field.

b) the distribution and magnitude of the stress components σ_r, $\tau_{r\theta}$ and τ_{rz} induced by the inclusion within the anisotropic body seem to depend upon the high or low modulus character of the inclusion, its geometry and the orientation of the anisotropy. In particular, the radial stress σ_r can be as large as the applied stress field along the external contour of a thick walled and high modulus inclusion. It decays rapidly within the anisotropic body but is still non negligible at a distance of five times the external radius of the inclusion unless the ratio of the outer to inner radii of the inclusion is small (less than 2) and/or a low modulus inclusion is considered. For a transversely isotropic body, the influence of the anisotropy is found to be the largest when the plane of transverse isotropy strikes parallel to the inclusion and to decrease appreciably when it strikes perpendicular to the inclusion.

The following parameters influence the stress distributions within and around an isotropic inclusion in an infinite anisotropic medium and must be investigated further:

- type and degree of anisotropy,
- orientation of the inclusion with respect to the principal applied stress field.

In Chapter 6, the closed form solutions derived in the previous chapters are used to solve the inverse problem, that is to calculate the principal components of a 3D in situ stress field and their orientation with respect to a fixed arbitrary global coordinate system, from measurements of strains and displacements associated with overcoring in one or several boreholes. No restrictions are made on the type or the orientation of the anisotropy or the orientation of the principal in situ stress field with respect to the different boreholes. The proposed theory is general and is applicable to the following types of overcoring measurements in anisotropic rocks:

1) changes in diameter of a pilot hole and changes in strain recorded on the walls of that hole,

2) changes in length between two points located in different cross sections at the inner surface of a hollow inclusion or within

a solid inclusion. The inclusion is perfectly bonded to the walls of a pilot hole. These changes in length reduce to changes in diameter when the two points are located within the same cross section. Case 1) is then a limiting case when there is no inclusion,

3) strain measurements using strain rosettes embedded at any point and in any direction within a hollow or solid inclusion perfectly bonded to the walls of a pilot hole,

4) combination of 2) and 3).

The advantages of the types of measurements in 2) and 3) with respect to those in 1) are such that the measurements are less affected by rock heterogeneities, discontinuities or wall irregularities. Measurements can also take place in weathered or jointed rocks and no surface preparation of the walls of the pilot hole is required. However, based on the results of Chapter 5, it is found that the only limitation is that low modulus thin walled hollow inclusions or very low modulus solid inclusions must be used for two reasons:

- to reduce the effect of ignoring the size of the overcoring diameter,
- to reduce the tensile stresses at the contact between the rock and the inclusion, induced by the inclusion during overcoring.

The proposed theory is also applicable for the determination of the components of any change of the in situ state of stress using the types of measurements in 1) to 4). In this case, the limitation previously mentioned does not apply.

Six independent measurements are required to calculate the six components of the in situ stress field. The following answers to questions (i) and (iv) of Chapter 1 are proposed:

a) in a single borehole there are at most three independent measurements of diameter and five independent measurements of change in length of oblique distances recorded at the inner surface of a hollow inclusion, or at the walls of a pilot hole. It is possible to obtain six independent measurements of strain by orienting strain rosettes in different directions.

b) the minimum number of boreholes required to calculate the complete state of stress depends primarily on the type of measurement used. Additional boreholes can always be used to increase the accuracy of the calculated stress field components. If strain measurements are used, one borehole may be enough. One borehole is also enough if measurements of change in diameter and change in length of oblique distances are combined. When changes in diameter are measured only, at least two non parallel boreholes are needed. The requirement for a third non parallel borehole depends upon the angle between the two boreholes, the isotropic, anisotropic character of the rock and the orientation of both boreholes with respect to the plane and axes of elastic symmetry of the rock. Whenever those conditions are such that two boreholes can be used, it is found that attention must be paid to the degree of anisotropy of the rock. The use of two boreholes is limited to moderately or strongly anisotropic rocks. For weakly anisotropic rocks (almost isotropic) the use of two boreholes produces erroneous answers and a third borehole is required.

c) since it is difficult to draw general conclusions for all types of anisotropy, questions (iii) and (iv) of Chapter 1 are answered for the transversely isotropic case only. It appears that neglecting anisotropy by assuming that the rock is isotropic can create large errors in both the magnitude and the orientation of the in situ stress field. An example involving the measurements of change in strain within a single borehole has shown that for low values of the strike angle of the plane of transverse isotropy with respect to the hole axis and for moderately or strongly anisotropic rocks, the errors can be as large as 50 to 80% in the magnitude of the calculated principal stress field. Its orientation can be up to 100 degrees off from the isotropic solution. This is found to be largely reduced if the hole strikes perpendicular to the plane of transverse isotropy or if the rock is weakly anisotropic.

The influence of rock anisotropy on stress measurements is not restricted to the special case considered (transverse isotropy). In more general symmetry classes (9, 13 or 21 elastic constants), the specific influence of anisotropy can be investigated for a given type and orientation of anisotropy using the computer programs developed here. This also applies to rocks whose anisotropy is associated with regular joint sets and are described as equivalent continuous media as long as the joint set stiffnesses are constant.

The results of this study suggest that the influence of rock anisotropy must also be investigated in the interpretation of measurements obtained from other techniques such as "hydraulic fracturing".

As far as "undercoring" is concerned, general formulas for the influence of rock anisotropy are presented in Chapter 6 and their application should be investigated further.