A Back Analysis Program System for Geomechanics Applications

by

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Summary of the Thesis

A Back Analysis Program System has been developed for geomechanics applications. The system inherits the developments made in the last two decades and adds new features to establish a multi-purpose back analysis program system, CADAX, to account for elasticity and elasto-plasticity both from deterministic and stochastic viewpoints, with a special interest in the quantitative evaluation of measurement system. The following statements give comprehensive summaries of the work described and problems identified in each chapter.

Chapter 1

Introduction to Back Analysis

Since the influential work of earlier times, back analysis has experienced a steady development resulting in various approaches where the differences arise from material modelling, algorithm structures, numbers and characteristics of unknown parameters to be determined, treatment of measurement data, consideration of uncertainties involved in estimation, and so on. In order to clearly define the intention of this thesis and identify the background of *"A Back Analysis Program System for Geomechanics Applications"*, this chapter firstly presents an overview of the various back analysis approaches considered in this thesis together with the conceptual evolution which supported the development of those approaches. Following this fundamental identification of back analysis methods, a detailed chronological review is made to identify several key researches in back analysis and to trace the branches of developments with time. This is followed by a classified review in which the developments identified with time are re-examined while identifying the characteristics, advantages and disadvantages of the previous work. Finally, various aspects of back analysis which are identified as those areas to be improved are summarized and the objectives of the thesis,

> *"Establishment of a sound basis of back analysis techniques and proposition of some new ideas for elastic and elasto-plastic back analysis procedures in deterministic and stochastic methods with a special interest in the quantitative evaluation of measurement systems"*

are stated in detail. Note that of the two points of importance in applying numerical analysis, namely (1) what model to assume, and (2) what parameters to use for the assumed model, this thesis places the main emphasis on the second point in which back analysis strategies including algorithm selection, measurement planning, etc. are discussed using relatively simple material models.

Chapter 2

Fundamental Back Analysis in Elasticity

Fundamental concepts of back analysis in elasticity are briefly introduced using the simplest structural problem depicted in Figure 1 to identify three basic approaches employed to back-calculate (1) an elastic constant, (2) an applied load, and (3) both of them, respectively. These fundamental concepts of back analysis are simple, yet important because all the subsequent developments are based on them.

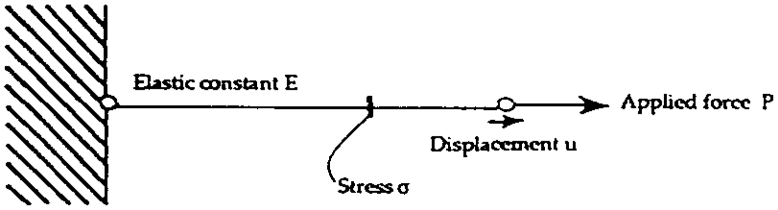

Figure 1 Analysis of a two-parameter system

Firstly, the characterization of Young's moduli (*Unknown: M of Material*) is discussed in the context of the linear elasticity. The *Direct Modulus Factoring Method* (DMFM) is introduced as an alternative approach to determine Young's moduli of multiple material zones under known load conditions. It is shown that the DMFM performs considerably better than the previously developed approaches for a class of problems in which the stress distribution has little dependence on the layout of material zones under the given load condition.

Secondly, the technique of back-calculating load parameters (*Unknown: L of Load*) from measured displacements and/or stresses, which is termed as *Direct Back Analysis Program* (DBAP), is discussed briefly in the context of linear elasticity. Several techniques are shown to properly subdivide a total load system into sub-load systems using various examples.

Thirdly, an algorithm, named *Double Back analysis of Material and Load* (DBML), for the simultaneous back analysis of Young's moduli and load parameters (*Unknown: ML*) is presented. The essence of this method lies in its double phase algorithm structure. Firstly, stress measurements are used to define load parameters and secondly these results are used with displacement measurements to define Young's moduli. Because of this structure, the characteristics and sensitivities of the method to various problem environments are inherited from its two component procedures DBAP and DMFM.

These three algorithms cover the first three unknown data categories and form the foundation of the back analysis system discussed in this thesis.

Chapter 3

Fundamental Back Analysis in Elasto-Plasticity

The three fundamental cases of back analysis discussed in Chapter 2, namely, back analysis of;

(1) material parameters,
(2) load parameters, and
(3) material and load parameters simultaneously,

are now considered in the context of elasto-plasticity. By performing back analyses using a common benchmark analysis, characteristics of each algorithm and measurement requirements for obtaining solutions are identified.

Firstly, it is shown that DMFM/P (an elasto-plastic version of DMFM) performs reasonably well for wide ranges of nonlinearities by using a relatively small number of displacement measurements. The conceptual strategy employed by this method is illustrated in Figure 2.

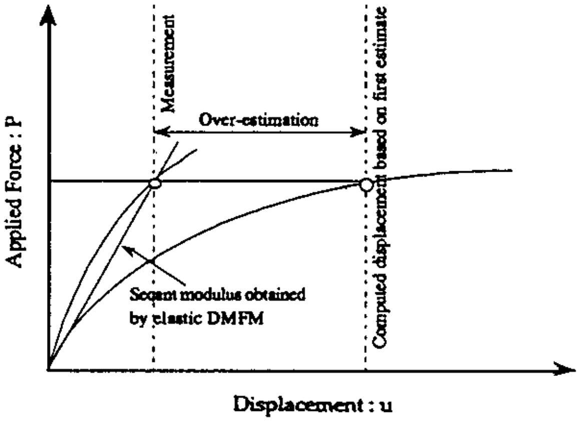

Figure 2 Algorithm strategy employed by DMFM/P

Secondly, in the application of DBAP/P, it is difficult to configure an appropriate displacement measurement pattern which ensures convergence, if the nonlinearity is severer than a certain level. Failures caused by using displacement measurements are mainly because of extreme first estimates which are unfavourable for the following plasticity analysis. Use of stress measurements can produce better results. However, a considerable number of stress measurement data must be used to obtain solution. Figure 3 shows the characteristics of this algorithm.

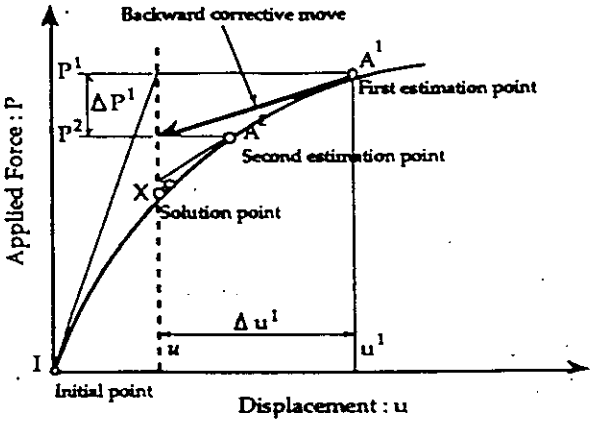

Figure 3 Algorithm strategy employed by DBAP/P

Thirdly, DBML/P is tested by using combined measurements of displacements and stresses for three cases of nonlinearity. The minimum numbers of measurements are found to be sufficient for the case exhibiting a limited extent of plasticity. For the severer cases, the measurement configurations involving unrealistically large number of measurements have to be used to achieve the convergence required to determine three load parameters and three Young's moduli. In the execution of DBML/P, the first component module DBAP/P plays a crucial role as the procedure starter. If the starting estimates, based on initial default values of Young's moduli, are inappropriate, DBML/P cannot converge to the solution. To produce appropriate first estimates of load parameters while Young's moduli are still incorrect, a considerable number of stress measurements have to be used.

The results obtained from the numerical examples reported in this chapter suggest that (1) the fundamental algorithms developed in elasticity are readily applicable in elasto-plasticity provided sufficient measurements are taken and (2) the most difficult task in back analysis in elasto-plasticity is to establish an economical method of characterizing load parameters in a severely nonlinear problem.

The six back analysis procedures presented in Chapters 2 and 3 belong, from the viewpoint of algorithm structures, to a class of *ad hoc* back analysis methods which are illustrated in Figure 4.

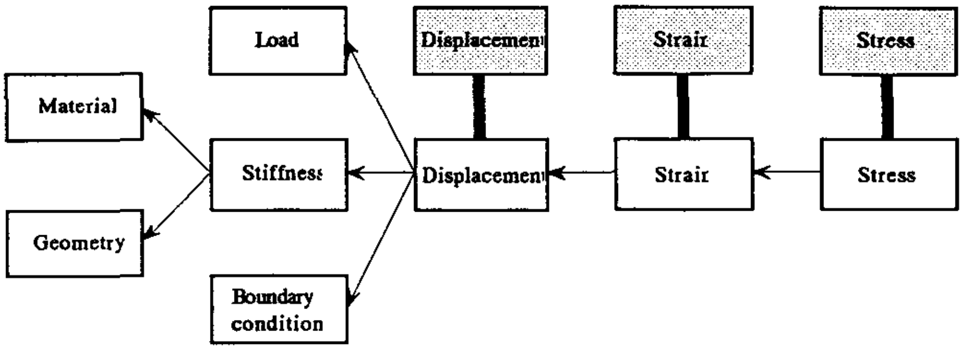

Figure 4 Ad hoc back analysis

Chapter 4

Direct Search Method

The fundamental back analysis algorithms discussed in Chapters 2 for elasticity and 3 for elasto-plasticity are specially designed for characterization of Young's moduli and load parameters, either independently or simultaneously. Of the seven types of unknown data category combinations, the first three,

(1) M (Material),
(2) L (Load),
(3) ML (Material and Load),

are covered by those back analysis algorithms. Note that at this stage, the category code M is used to refer to only Young's moduli. If an unknown data category combination was of other than these three types, or if material parameters other than Young's moduli were included in a list of unknowns, a more general solution method is required, such as the *Direct Search Method* (DSM). In this method, a suitable mathematical function minimization tool is coupled with a forward or backward analysis procedure in order to create an automatic parameter modification loop based on field measurements. In this general tool, virtually any quantity involved in an analysis procedure may be chosen as the unknown parameter and optimally determined such that the discrepancy between computed results and measurements is minimized. Consequently, the DSM becomes an ideal procedure for application to the other four types of unknown data category combination, namely

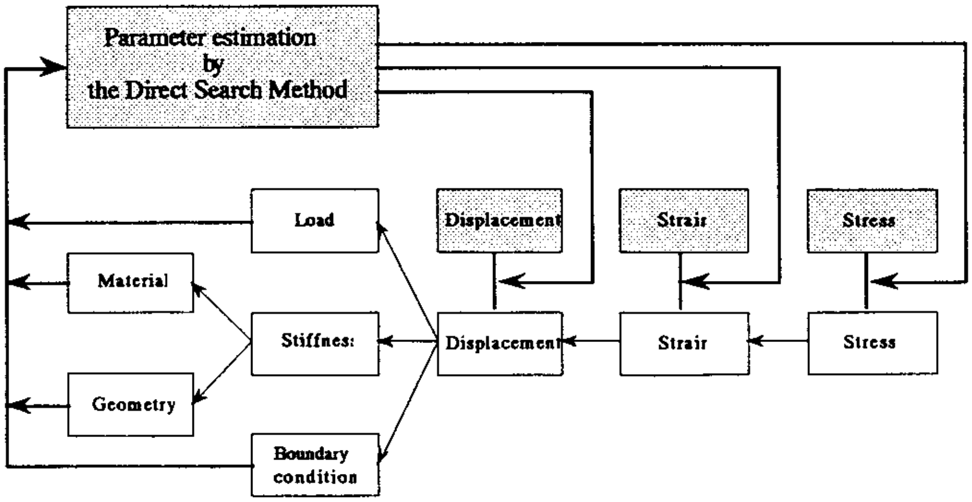

Figure 5 Backward direct search method

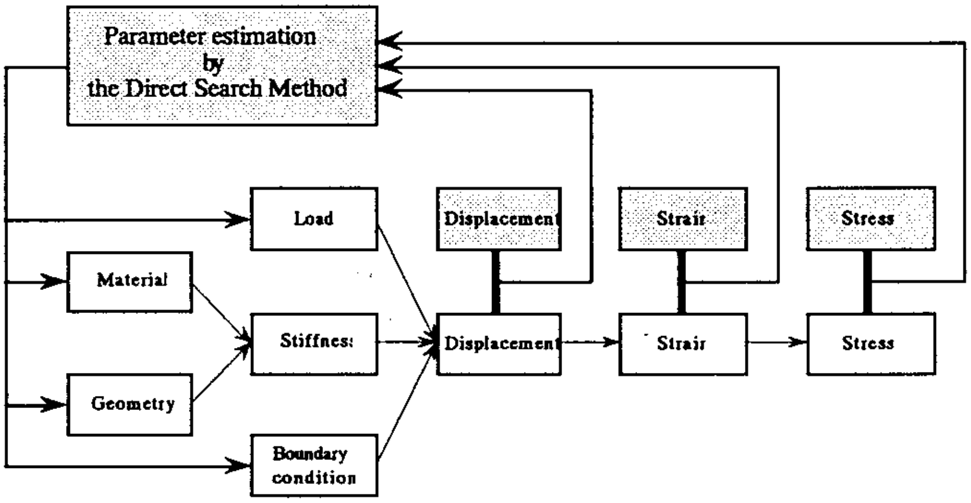

Figure 6 Forward direct search method

(4) G
(5) GM
(6) GL
(7) GML

where G stands for *Geometry*. The thesis employs Rosenbrock's method as a mathematical function minimizer which is coupled with the six back analysis algorithms (DMFM, DBAP, DBML, DMFM/P, DBAP/P, and DBML/P) forming *backward DSM* and the two forward analysis modules (FEM and FEM/P) forming forward DSM. The abbreviated forms of these procedures are:

ROSENBROCK(FEM)
ROSENBROCK(DMFM)
ROSENBROCK(DBAP)
ROSENBROCK(DBML)
ROSENBROCK(FEM/P)
ROSENBROCK(DMFM/P)
ROSENBROCK(DBAP/P)
ROSENBROCK(DBML/P)

and their conceptual algorithm structures are illustrated in Figures 5 and 6, respectively. Benchmark analyses reveal that problems of all unknown data category types in elasticity and in elasto-plasticity are readily solvable by these Direct Search algorithms. Despite the generality of the concept, however, the practical performance of this method is affected by many factors, such as the selection of measurements, management of search strategy, etc. As the number of unknown parameters becomes greater and the analysis type more complex, the practical costs of using this option may become prohibitive and the benefit minimal.

Chapter 5

Stochastic Back Analysis and Measurement Grade System

In this chapter, firstly a review is made as to the fundamental concept of stochastic parameter estimation whose essential characteristics lie in the use of information associated with the randomness of measurements, and the parameters being estimated and, in some cases, with knowledge of the statistical properties of the parameters known prior to estimation analysis.

Secondly, a general form of the iterative back analysis algorithm based on Bayesian method is reintroduced. In the application of the Bayes' method, a slight modification is made to the formulation developed previously so that the measurement data may be *displacements*, *stresses*, or *both*. The magnitudes of the perturbations in each iteration are also modified so that perturbation size is automatically reduced as the updated estimate improves.

Thirdly, Bayesian methods are formulated by using back analysis modules as the core algorithm to be repeatedly called within the frame of an iterative back analysis. Restricting discussion to elasticity, those hybrid algorithms to be discussed are, by using a similar notation to that used in Chapter 4,

BAYES(DMFM)
BAYES(DBAP)

BAYES(DBML)

By formulating these algorithms, it is shown that construction of a stochastic back analysis procedure which includes both deterministic and stochastic modules as its structural components, is feasible, as illustrated in Figures 7 and 8.

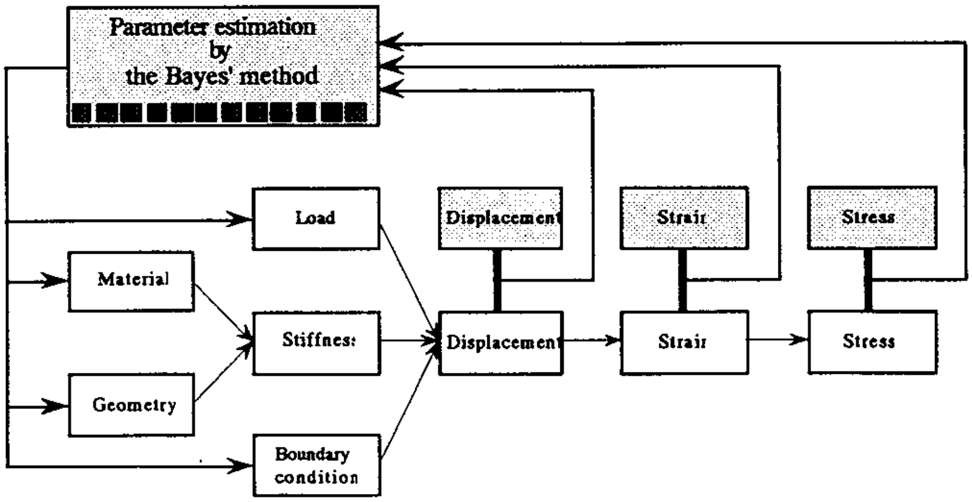

Figure 7 Forward Bayes' method

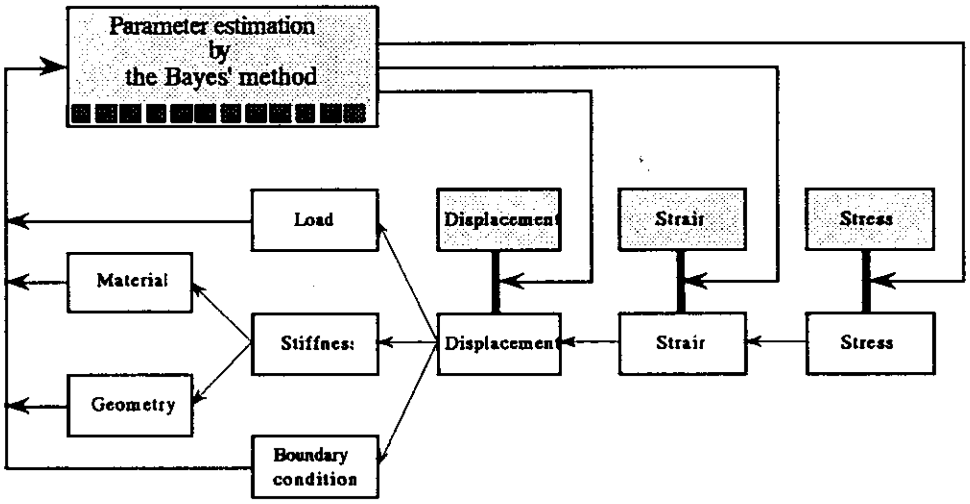

Figure 8 Backward Bayes' method

For such an algorithm, selective use of measurement data is shown to be effective in limited circumstances. Numerical examples dealt with in this section are also configured such that the effects of different measurement patterns on solution accuracy can be observed.

The latter half of this chapter presents a turning point in the thesis. The discussions up to this point has been as to *IF* and *HOW* unknown parameters may be back-calculated under various circumstances. Now, with the help of the Bayesian back analysis method, the main interests shift towards *HOW ACCURATELY* and *HOW EFFICIENTLY* parameters can be estimated. The efficiency of a parameter estimation process and the accuracy of solutions are dependent upon various attributes involved in a decision process of a measurement configuration, as pointed out on a number of occasions in the thesis. For example, the following eight attributes may be considered to be of practical importance:

(1) Accuracy of measurements
(2) Accuracy of initial estimates
(3) Location of measurements
(4) Direction of measurements
(5) Distance between measurement points
(6) Components of measured quantities
(7) Type of measured quantity
(8) Number of measurements

By changing these attributes, solutions are obtained with different accuracies even within the same problem. Therefore, by comparing the accuracies of solutions obtained from different measurement configurations, their relative efficiencies may be evaluated. When the problem has only one unknown parameter, the percentage standard deviation (PSD: a square root of an arbitrary diagonal term of a posterior covariance matrix, representing the standard deviation of estimation error for a given parameter) may be used as a reference index to assess measurement efficiency. For a case in which there is more than one unknown parameter, say N_p, PSDs for all parameters may simply be averaged to compute a single percentage number which represents the efficiency of the measurement system employed for that particular problem. This number is called the *Measurement Grade* (MG) and is defined as

$$\mathrm{MG} = \frac{1}{N_p} \sum_{i=1}^{N_p} \mathrm{PSD}_i$$

A brief study on the fundamental conditions required to obtain solutions with higher accuracies show that efficient measurements generally possess greater sensitivities to parameter perturbation than inefficient measurements. A greater number of measurements also usually results in a good estimation, provided the measurements are of reasonable quality. The essential task of planning an effective measurement configuration is to install a sufficient number of sensitive measurements to satisfy design criteria. Selection of sensitive measurements includes consideration of many factors such as location, span length, direction, component, etc. These matters need to be carefully organized to maximize the efficiency of the measurements under a given loading condition. In addition to the list of attributes mentioned previously, the practical decision process of the measurement system would include consideration of factors such as

(1) Economy
(2) Access
(3) Risk
(4) Parameter dependent design accuracy criteria
(5) Coordination of devices with different accuracies
(6) Durability of instruments
(7) Purpose of a device (whether it is made to measure contraction or elongation)

Some of these factors will be discussed in Chapters 6 and 7 where applied numerical examples will be considered. Lastly, an alternative method is proposed for computing Measurement Grades quickly using the path-independence of the computational procedure of the covariance matrix $\mathbb{C}_P$. This method is referred to as the *Quick Co-Variance (QCV) method* and used frequently in the subsequent chapters.

Chapter 6

Applied Back Analysis in Elasticity

In this chapter, the applied use of fundamental back analysis procedures is discussed in elasticity for selected algorithms with a special attention to design of efficient measurement configurations.

Firstly, the performance of the Direct Modulus Factoring Method was compared with those of Rosenbrock's and Bayes' methods in various problem configurations. The DMFM performs with the least computational effort, but its convergence stability is dependent on the problem configuration. The Bayes' method requires N_p (which is the number of unknown parameters) times the numerical effort required by DMFM, but assures more stable solutions. Rosenbrock's method requires even greater computational effort while its performance is not as stable as that of the Bayes' method. According to these findings, the DMFM may be employed for problems in which the configuration of material zones and load conditions is in favour for the DMFM, especially if the problem has to be solved a number of times in an analysis project. If the number of back analyses required to complete the project is limited and the Young's moduli are to be estimated stochastically, the Bayes' method is an ideal option.

Secondly, the DBAP algorithm is extended to the new algorithm DBAP/Multiple-Stage as the only procedure, as illustrated in Figure 9, which involves *more than one load application process in a single back analysis formulation.* By this new algorithm, the in-situ stresses may be determined by measurements taken during an arbitrary stage of a multiple stage sequential excavation. However, in practice accuracies of the estimated in-situ stresses vary considerably depending on the quality of the measurement data. Sensitivity analyses such as those conducted in this chapter using the Measurement Grade System and the QCV method helps determine recommended measurement patterns bases on prescribed design criteria.

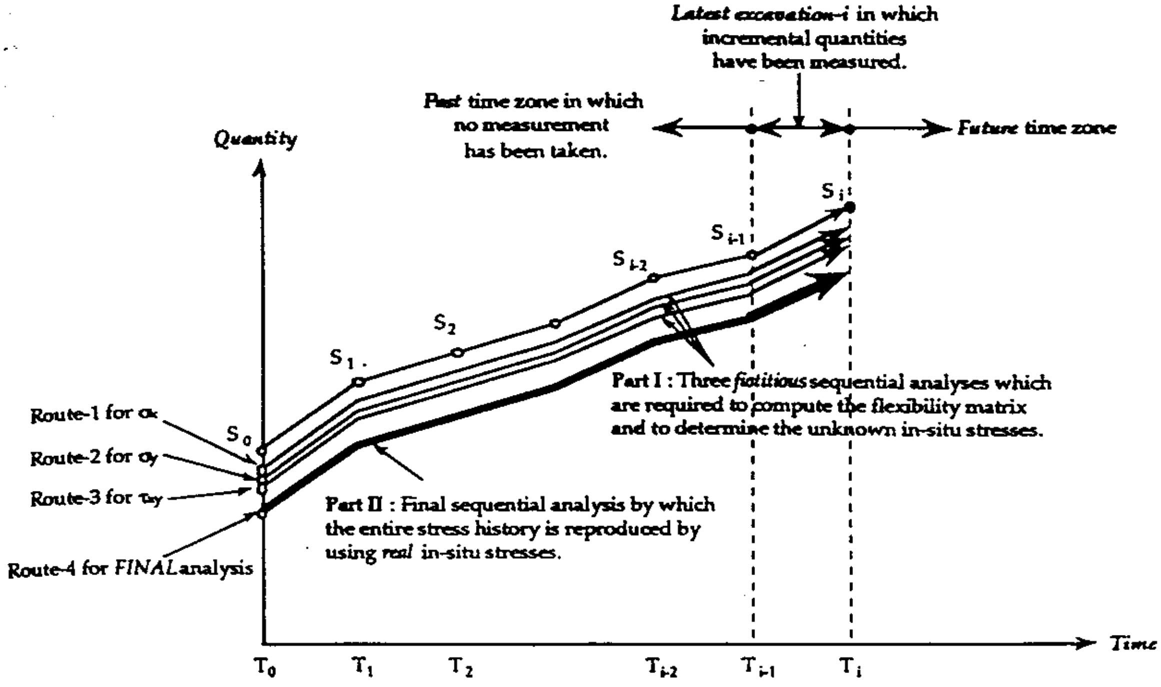

Figure 9 Graphic illustration of the algorithm DBAP/Multiple-Stage

Thirdly, the hybrid stochastic algorithm BAYES(DBAP/Multiple-Stage) was employed to estimate the unknown parameters in categories M and L in a multiple stage excavation problem in search of the best stress measurement location for the highest solution accuracy (see Figure 10). This method might be of practical interest because there are many occasions in which the rock structure's safety is to be investigated whereas neither Young's moduli nor the virgin stresses are known after several stages of excavation. However, this method also is crucially dependent on the use of the principle of superposition possible only in linear elasticity.

In the second and third examples, simulations of the process of determining recommended measurement configurations are performed, using the Measurement Grade System and the QCV method. When the unknowns are confined to one data category as in the second example, the fundamental strategy of measurement planning is relatively straightforward. If the measurements are to be performed over a period of time during which several excavations are conducted at different locations, they have to be planned to produce balanced results for all events.

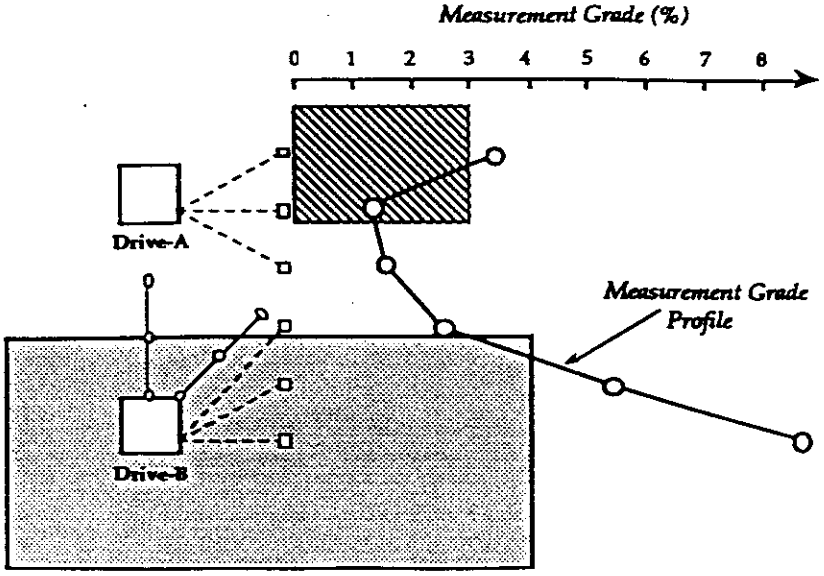

Figure 10 Measurement Grades for six patterns

Measurement Grade (an average standard deviation, in percentage, of errors of estimated parameters) becomes the smallest for the stress measurement point accessed from Drive-A. An engineer can easily decide from this diagram that a stress measurement be performed where the value of MG becomes the lowest.

On the other hand when the unknown parameters are in two complementary categories, namely the Young's moduli and load parameters, care must be taken to tune the measurement configuration both in terms of what quantity is measured and where it should be measured, especially when the number of unknown Young's moduli is greater than 1. A generally recommended practice is that both stresses and displacements should be measured to be used selectively for the estimation of the load parameters and Young's moduli, respectively. To obtain solutions with higher accuracies, stresses should be measured at locations where sensitivities of stress distributions to load parameters and to changes in Young's moduli are maximum and minimum, respectively. Similarly, displacements should be measured at locations where sensitivities of the displacement field to changes in Young's moduli and to incorrectly computed load parameters are maximum and minimum, respectively. This indicates that the stresses and displacements should be measured at different locations, each of which is suitable for the estimation of the respective unknown parameters. If, for the estimation of in-situ stresses and *more than one* Young's modulus, an access were limited and both quantities had to be measured more or less in the same region, one must therefore expect that accuracies of the estimated parameters may not be as high as they could otherwise have been.

Chapter 7

Applied Back Analysis in Elasto-Plasticity

In this chapter, the applied use of fundamental back analysis procedures in elasto-plasticity is discussed for selected problems in three sections. Interests are again placed on the appropriate configuration of measurement patterns in various problems encountered.

Firstly, the Measurement Grade System and the QCV method are applied to investigate appropriate measurement strategies to estimate the cohesion and friction angle of a Mohr-Coulomb material. In this investigation, appropriate measurement orientations, locations, and quantity-type are studied in association with the principal stress directions and the extent of plastic deformation from a localized viewpoint. As in the estimation of any parameters such as in-situ stresses, Young's moduli, etc., the results of the investigation show that there are particular directions of measurements which produce more accurate estimates of Mohr-Coulomb parameters than other directions, see Figure 11.

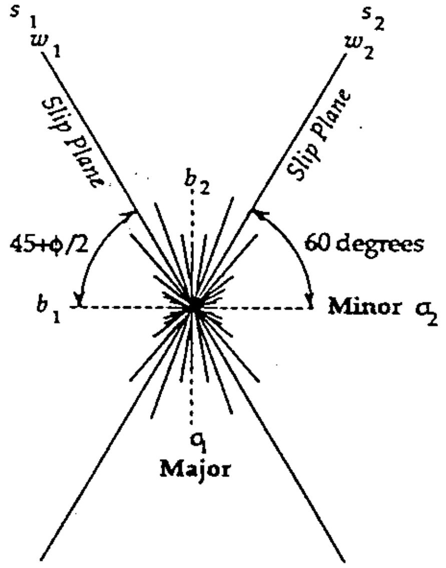

Figure 11 Measurement Grades evaluated for the estimation of cohesion

Magnitudes of Measurement Grade are plotted along the direction of displacement measurement. The shorter the plotted line is, the more accurate the estimated cohesion. This diagram shows clearly that there is a preferred direction (namely, horizontal direction) which produces high quality displacement measurement to be used for the estimation of cohesion.

If the state of stress were known for a measurement point, the best measurement direction may be explicitly defined for that particular point. However, on a global scale, a measurement line is usually greater than at least the dimensions of finite elements used around measurement locations. Consequently, there will be more than one Gauss point distributed over the length of the measurement device. If states of stress at these Gauss points were more or less the same, and therefore the principal stress directions were the same, then the findings made in this investigation may be effectively used, especially if displacements were used as measurements. Otherwise, if principal stress directions of the Gauss points were considerably different, the results obtained in this section may not be used directly, and direct evaluations of the Measurement Grades may become necessary.

Secondly, a fundamental study is conducted to establish a basic measurement strategy to characterize Mohr-Coulomb material properties of a discontinuity plane. When selecting the appropriate position of the measurement location over the length of a joint plane, highly stressed and actively deforming positions may be chosen taking into account of practical restrictions. If a joint plane were in opening mode in an only accessible area, there should not be a problem as long as that opening behaviour is closely influenced by the behaviour of an adjacent closed section of a joint, in which c and ϕ have active roles. No difference in measurement strategies is required for the estimation of c or ϕ, because MG profiles for those unknowns have similar characteristics.

Having identified fundamental features involved in the estimations of nonlinear material parameters of a continuum and a discontinuity plane, a final numerical simulation is presented to conclude the main body of the thesis. In this illustration, an ideally configured numerical example is presented in which knowledge of linear and nonlinear material properties of a rock mass and a fault plane and in-situ stresses are accumulated progressively in a staged excavation problem, as depicted in Figure 12.

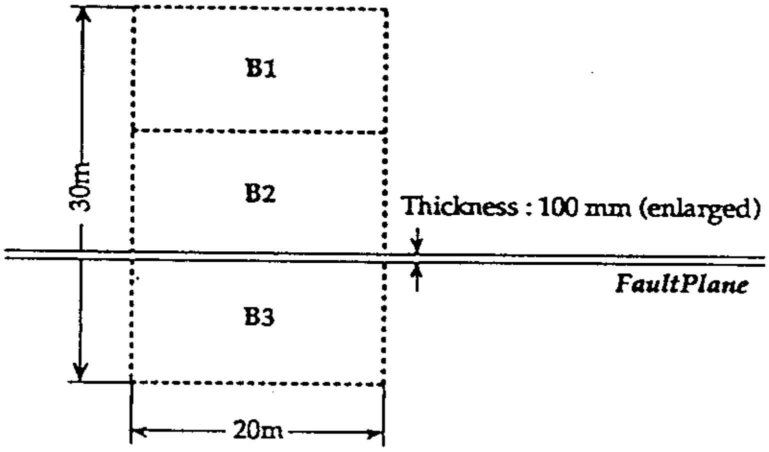

Figure 12 Construction site of an underground cavern

This problem is configured such that characterization analyses are conducted in three phases. In the first elastic phase, the algorithm BAYES(DBAP/Multiple-Stage) is used to estimate in-situ stresses and elastic constants of a rock mass during a small sized excavation. In the second elasto-plastic phase, a larger volume of rock is excavated, which results in part of the rock mass behaving elasto-plastically. In this phase, Mohr-Coulomb parameters of the rock mass are characterized by the algorithm BAYES(FEM/P). In the third phase, a further excavation is performed which causes a major discontinuity plane present in the domain to open and slide, during which its Mohr-Coulomb parameters are identified by the algorithm BAYES(FEM/P). In each phase, Measurement Grade and the QCV method are used to optimally design measurement configurations. An example of this is shown in Figures 13 and 14 which illustrate the design strategy and Measurement Grades computed for candidate measurement locations.

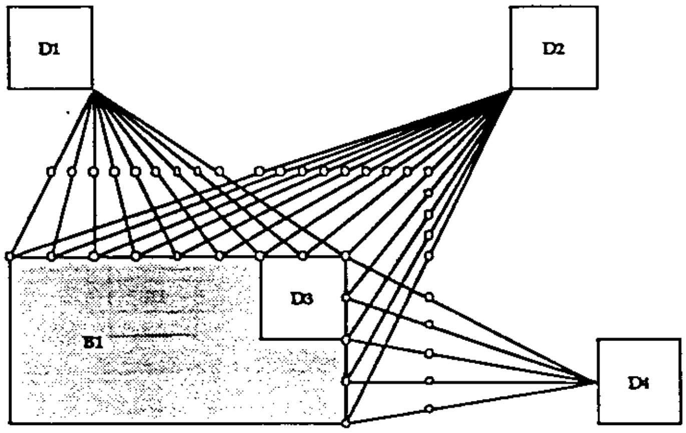

Figure 13 Extensometer locations tested for the estimation of c and ϕ

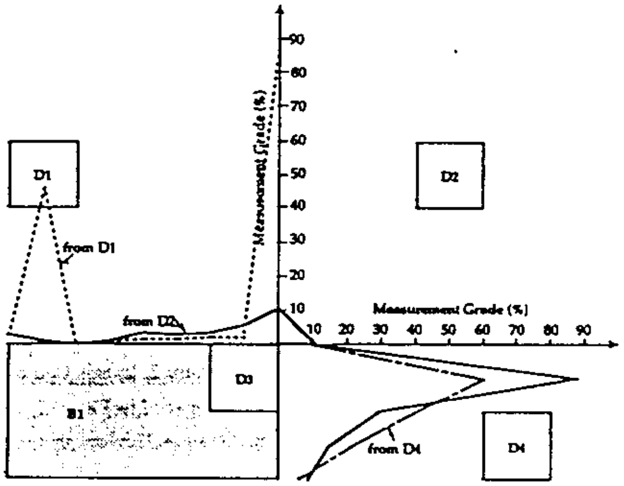

Figure 14 MG plot for estimating c

At the end of the third phase, a complete set of knowledge of the rock mass and the in-situ conditions becomes available which may be used effectively in predictive analyses to simulate the final excavation phase. Throughout characterization analyses conducted in the three phases, the MGS and the QCV method are used to design appropriate measurement patterns, the summary of which is shown in Figure 15, using the findings of this thesis. Though it may have been too ideally configured, this example shows a typical procedure involved in the decision making process to monitor a staged excavation. It is shown that in each phase the applications of the MGS and the QCV method are useful in designing effective measurement patterns and determining the accuracies of the parameter estimates that those measurement patterns would produce.

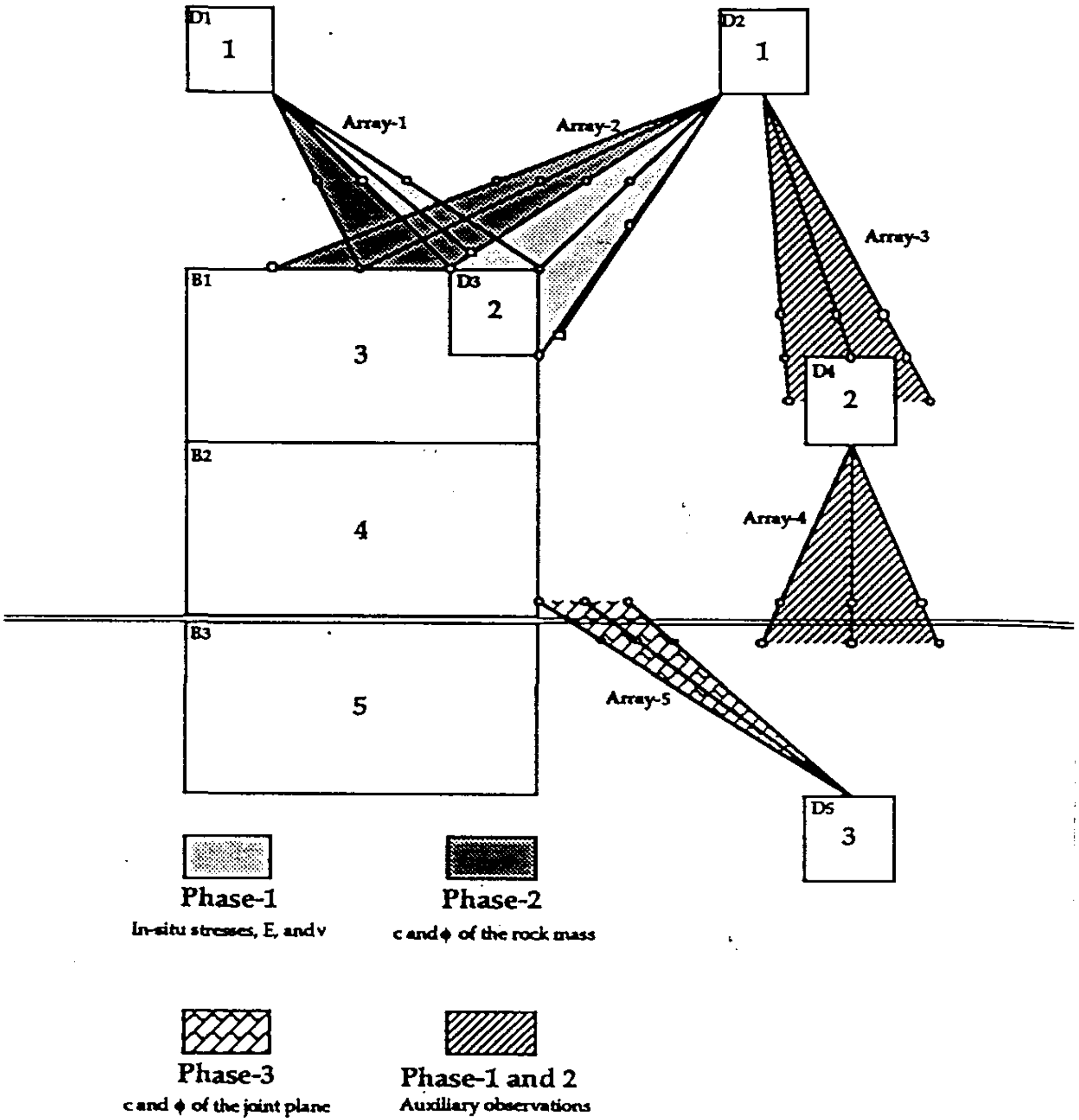

Figure 15 Summary of measurement strategies for the three phases

Chapter 8

Conclusions

The state of back analysis in geomechanics is graphically represented to conclude the presentation and clearly identify areas of interest for further research. In this illustration, the classification scheme described in Chapter 1 is used to categorize various factors considered in back analysis. For each category, the previous developments introduced in Chapter 1, the contributions made in this thesis, and further research interests are identified. In the final classified review, graphic representations are made by which the nature of the work considered is categorized. Firstly, rectangular cells with round corners indicate category specifiers. Qualifiers in normal rectangular cells which branch out of these category specifiers indicate elements of work considered in back analysis applications. A qualifier (or work) which has been accomplished by previous researchers has normal letters in a white cell. The contributions made in this thesis are indicated by *italic letters* in shaded cells. Those cases which have been developed previously and discussed in this thesis again are indicated by a shaded cell with normal letters. Areas of further research interest are emphasized by ***outlined italic letters***. If a cell with outlined letters is white, it indicates a new research topic to be explored. If that cell is shaded, it indicates an area which has already been developed to a certain extent but requires further improvement. If a cell is shadowed by another black cell, it means that the particular application indicated by that cell has been applied to field problems and substantiated by actual measurements and back analysis procedures. These conventions are summarized in Figure 16.

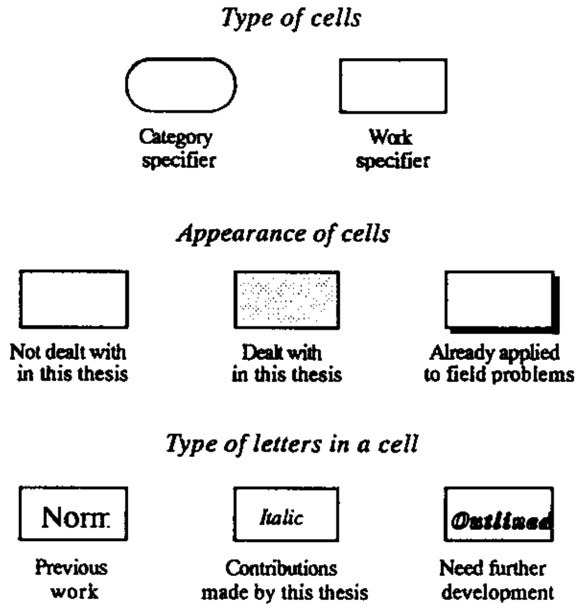

Figure 16 Conventions used to represent the state of back analysis

Geomechanics System

The state of back analysis associated with the specification of the *Geomechanics System* is shown in Figure 17. Most work presented thus far including this thesis have been concentrated on modelling of a continuous geometry. A limited number of discontinuous geometries having a joint plane have been discussed in this thesis. However, these presentations were greatly simplified compared with what actually exists in the field. The explicit modelling of blocky rock structures has not been formulated in terms of back analysis application. The use of *infinite* finite elements to model an unbounded geometry, especially for underground excavation problems, such as the examples discussed in Chapters 6 and 7, improves the discretization procedure. However, more accurate modelling of an infinite region may be performed with much less effort of discretization by a boundary element approach.

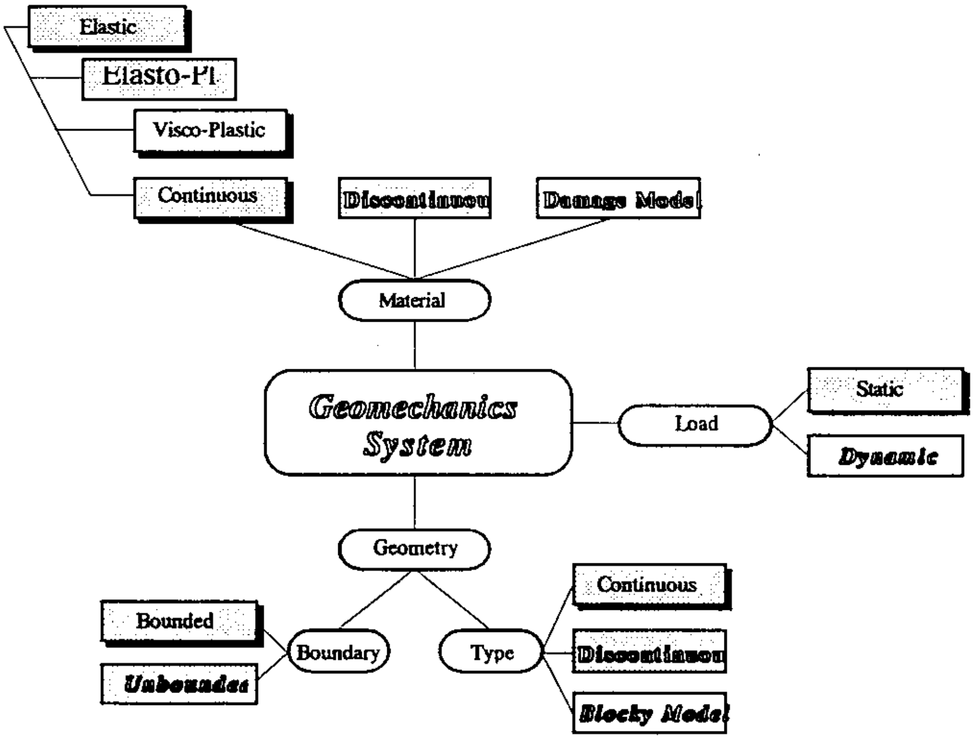

Figure 17 Geomechanics System

Coverage of back analysis procedures in elasticity and elasto-plasticity involving continuous materials has been fairly well documented so far by the use of general estimation methods. Relatively less developed areas are concerned with problems involving visco-plasticity and a damage model. Both of these areas require further improvement. Dynamic behaviour of displacements and stresses observed during blasting have not been accounted for from a viewpoint of back analysis. This may become one of the major areas to be addressed in the near future.

Event

Figure 18 shows the state of back analysis with regard to the specification of the *Event*. As mentioned earlier, the types of events which have been accounted for in back analysis applications are limited to static loading. In practice, excavation is conducted mostly by blasting. The actual behaviour of a rock mass during blasting is far more complex than that caused by a simple static loading. Because quantity changes measured in-situ are functions of all factors (blast dynamic shock, creation of new fractures, stress redistributions, block movements etc.), the explicit or implicit modelling of these phenomenon is required to characterize the actual behaviour and decompose quantity changes into several components which correspond to their respective causes.

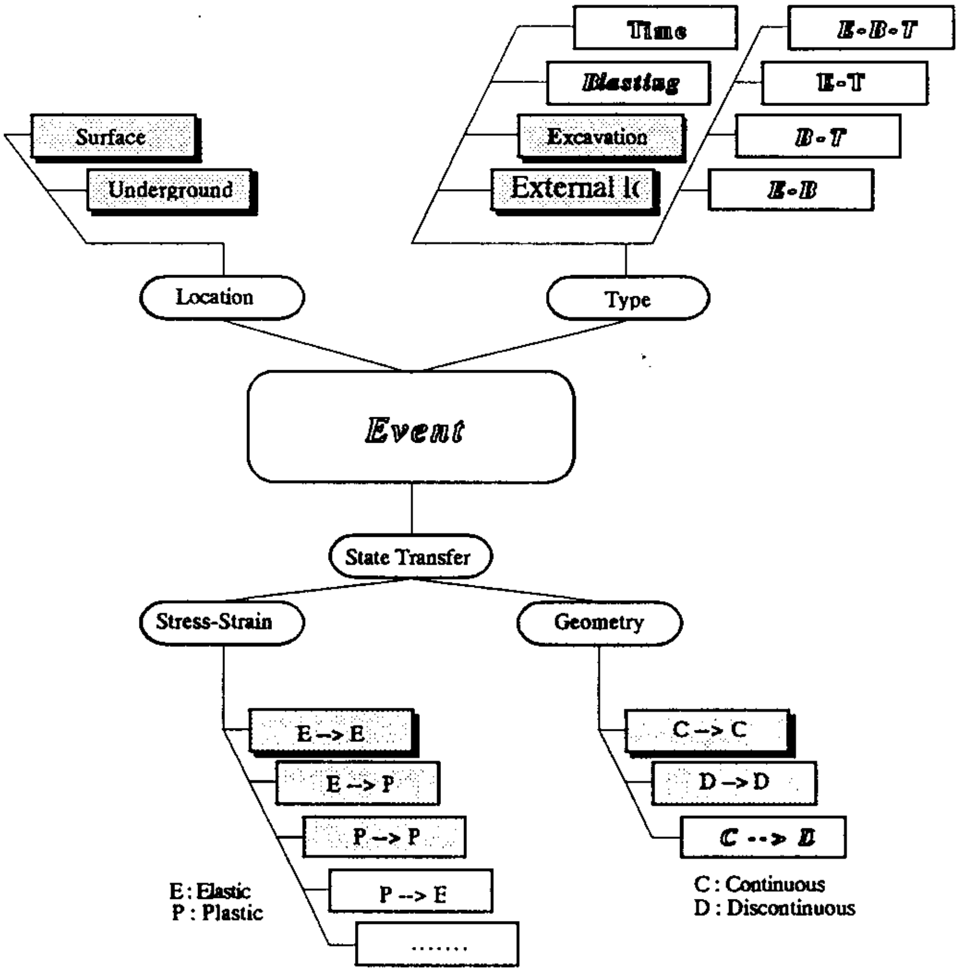

Figure 18 Event

Modelling of geometry changes associated with not only excavation but also the creation of new fractures which corresponds to the cell $\mathbb{C} \dashrightarrow \mathbb{D}$ on both small and large scales may also be accounted for by an explicit or implicit modelling of these behaviours. The reviews made with Figure 18 suggest that more rigorous modelling of the actual behaviour of rock masses is required both from the explicit and the implicit viewpoints.

Measurement System

In this thesis, the use of stress measurements along with displacement measurements was emphasized to ensure a stable solution for cases having unknown parameters both in material and load categories. Since the discussions have been limited to static problems, measurements were automatically restricted to displacements and stresses which are the two measurable quantities of practical significance. The extension of back analysis into dynamics could involve measurements of dynamic displacements and accelerations. For a balanced assessment of the stability of rock structures, observation of other quantities or the state of the rock mass such as the rock joint distribution, etc., could enhance the characterization procedure. Figure 19 shows an example of a measurement management sheet.

Measurement Management Sheet			**Type of Measurements**							⬇ Category Dependent Measurement Grade	⬇ Final Measurement Grade
			Displacement	Stress	Vibration	Gas Pressure	Joint Distribution	Bore Hole Image	Acoustic		
Measurement Pattern		Location									
		Resolution									
		Number									
		Source Medium									
		Accuracy									
Measurement Grade	*System Parameter*	Geometry	▨	▨							
		Material	▨	▨							
		Load	▨	▨							
	System Quantity	Displacement									
		Strain									
		Stress									

Figure 19 Measurement management sheet

Each column represents a different type of quantity or state of rock masses which could be measured or observed. The table is divided into two sections at the mid height. The upper section specifies the details of a measurement pattern. This section should provide explicit information on each attribute of a measurement system. The lower section may be used to summarize results of a back analysis which are obtained by using the measurements specified in the upper section. This section is further divided into two parts. In the first part, the results of characterization of system parameters are specified. An index to be specified in each cell should be able to define how accurately a particular parameter has been characterized by the measurements of that column. The Measurement Grade proposed in this thesis may be used for this purpose, if a back analysis procedure could be defined in statistical terms. In the second part, the results of characterization of system quantities are specified. For example, a cell of (Strain,Displacement) should provide an index or a set of indices which define the accuracy or credibility of a strain field obtained by back analysis using displacement measurements. The lower section therefore helps identify the contributions made by each measurement quantity to the accuracies of (1) system parameters and (2) system quantities computed from the characterized parameters. The various numerical examples presented in Chapters 6 and 7 may therefore be specified by a simplified version of this table in which hatched cells become active. At this stage, the Measurement Grade System is merely a scheme by which these six cells can be filled. However, it may be improved further such that all cells in Figure 19 may become active as a full scale Measurement Grade System.

Unknowns

Figure 20 illustrates the fundamental features involved in the specification of unknown parameters and possible combinations to be considered in elasticity, elasto-plasticity, and visco-plasticity.

The fundamental classification of unknown parameters using G, M and L may have to be updated to account for a more precise specification of unknown parameters, especially in the light of more rigorous modelling which is expected to follow in the near future. For cases involving optimization of geometric parameters, an improvement of an automatic mesh generation procedure may be beneficial.

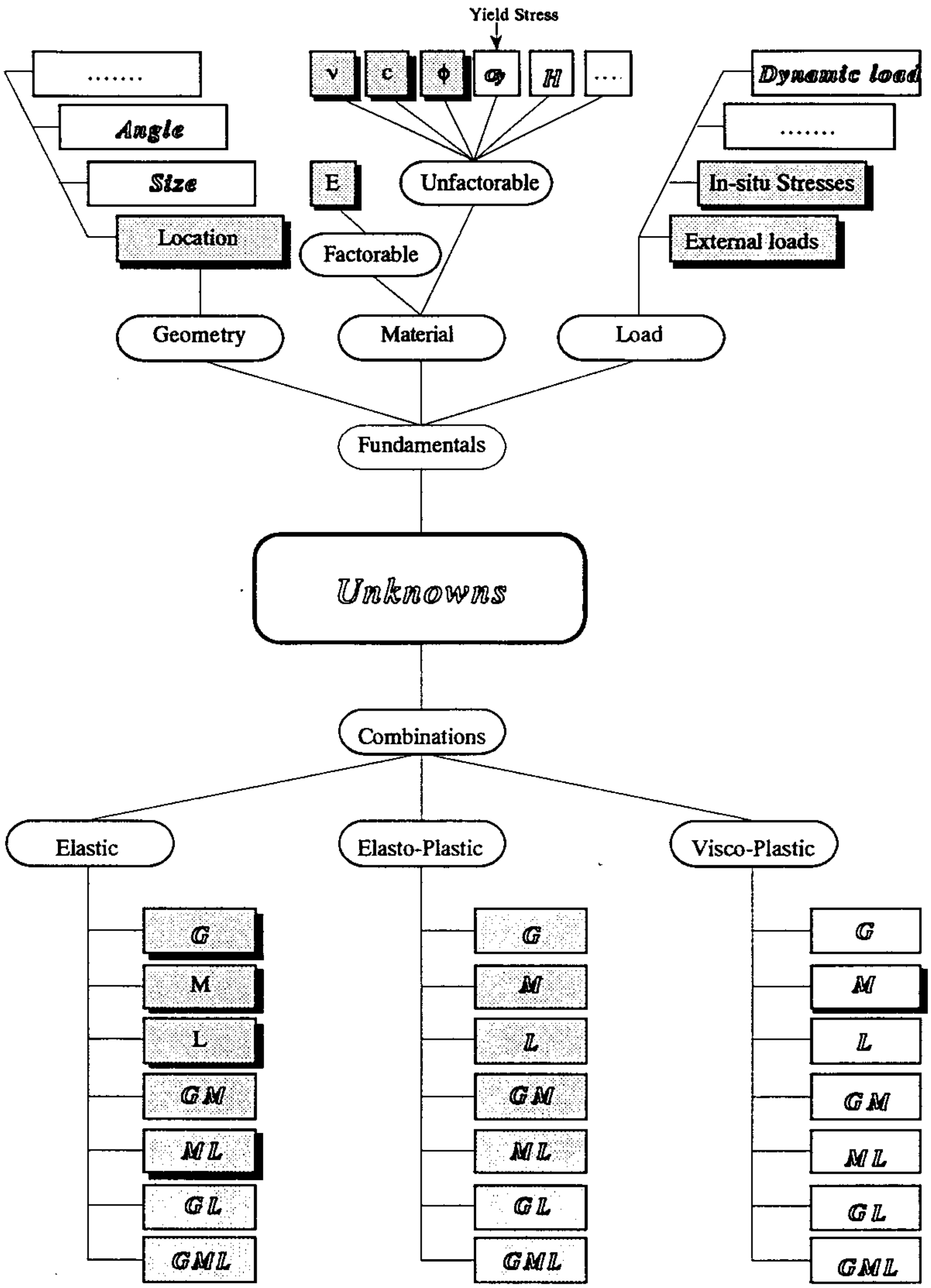

Figure 20 Unknowns

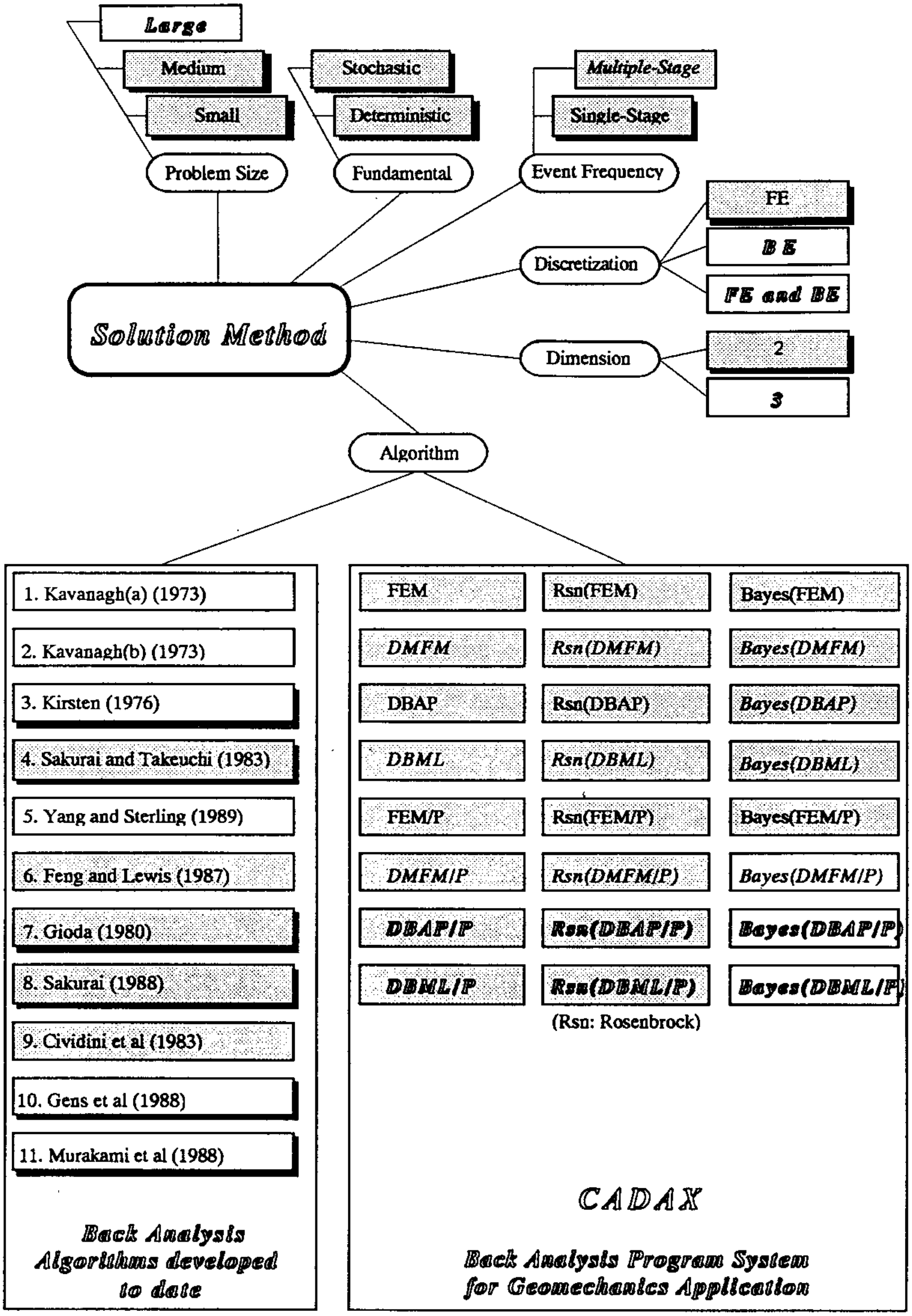

Figure 21 Solution Method

Solution Method

Figure 21 shows the current state of the solution methods used for back analysis in geomechanics. All types of unknown data categories are covered in elasticity and elasto-plasticity. However, there are many aspects of the available algorithms to be improved for more efficient estimation procedures. As mentioned earlier, the majority of the work conducted to date including that reported in this thesis considers the two-dimensional case. A thorough calibration of the back analysis procedures for the three-dimensional case is required. The algorithm DBAP/Multiple-Stage and its Direct Search and Bayes' versions has added another dimension to the library of back analysis procedures for elastic cases. However, its benefit can only be appreciated in a strictly elastic condition. An exploration of the application of a similar approach to nonlinear problems would be a worthwhile exercise.

For a back analysis program system to be of practical use, especially for three-dimensional problems, the incorporation of the Boundary Element Method is essential. The advantages of the BEM help improve various numerical processes associated with geometry modelling, treatment of unbounded regions, positioning of arbitrary measurement sampling points, etc. A combined use of the FEM and BEM would therefore be an ideal option.

The bottom half of Figure 21 shows the list of back analysis algorithms developed to date and these contributed by this thesis. Those algorithms shaded in the group of previous contributions indicate that the fundamental approaches taken by them are inherited by the current back analysis program system. Of the algorithms developed to date, about half have been applied to field problems, which are all formulated for the two-dimensional case. Extensive calibration is required for all back analysis procedures, especially for the three-dimensional case.

Directions for Further Research

The developments reported in this thesis have contributed to the establishment of the most fundamental form of a multi-purpose back analysis program package to account for the seven types of unknown data categories in elasticity and elasto-plasticity both from deterministic and stochastic viewpoints. This provides a start point from which a more general and practical form of a back analysis program system may be developed by incorporating the further developments mentioned in Section 8.3. In implementing these features into the existing program system, four directions of further research and development should be pursued.

More Versatile : Boundary Element Method and Dynamics

Firstly, the back analysis program system must become *more versatile.* Two major developments could be achieved by incorporating the Boundary Element Method and the dynamic behaviour of a rock mass into the solution. Along with these two major developments, more versatile material models need to be implemented to enhance the capability to describe the nature of a rock mass more effectively. An implicit approach to model rock joints, such as a damage mechanics theory, is of practical importance.

More Systematic : Integrated Measurement Management System

Secondly, a *more systematic* program package needs to be developed. A major factor involved is concerned with an efficient and practical method of designing measurement configurations. The example shown in this thesis using the Measurement Grade System and the QCV method offers a first step in this direction. The automation of this procedure using computer graphics would be beneficial for industrial applications.

More Practical : Application to Complex Excavation Problem

Thirdly, a program system must be developed to account for *more practical* applications. Though it may be theoretically complex, most of the previous work including that presented in this thesis deals with rather simple examples. In practice, there often exists a case in which the safety of a rock mass needs to be assessed for a particular site in, for example, a large scale underground mine. It is often the case that in these problems there are limited numbers of measurements available for back analysis purposes. The numbers of excavation stages preceding the first measurement may be in the order of 10 or even 100. The behaviour of rock masses is often nonlinear and the joints distributed within rock masses may control deformational behaviour. A large proportion of the measured displacements may be caused by direct blast shock. These descriptions of a typical underground excavation problem are not exaggerated. The question must then be asked as to whether or not the explicit modelling of these complex behaviours is of practical significance or even a possibility from the viewpoint of back analysis. *Ad hoc* implicit methods supported by empirically established methods may prove to be superior to an explicit approach in some cases. An extensive investigation is required to establish a methodology of applying field measurements and back analysis to these complex practical geomechanics problems.

More Efficient : CAD and Automation

Lastly, the back analysis procedure must be *more efficient* and easy to apply. More efficient algorithms, faster computational techniques, etc. are required. Above all, the most attractive direction of development for an efficient back analysis system involves the automation of the various procedures in the form of a Computer Aided Design system. Data preparation, measurement planning and graphic data processing may be implemented initially because they could be installed without the development of major solution algorithms. With an expectation of ever faster and large-capacity computer hardware, one may consider the assembly of all back analysis procedures in an integrated interactive computer program system.